PSYCHOPHYSICS

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By the same author

LIFEBALL

MINDWORLDS

G.O.D. IS GREAT

PHILOSOPHER

CORAL

BRITIZEN JON

ALBION

PSYCHOPHYSICS

A Brief Introduction
To Unfolding Reality

J. Andrew Ross

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PSYCHOPHYSICS

SCHILLER'S HAIKU

This great realm of souls:
its chalice foams and bubbles
to infinity.

INTRODUCTION

The human mind as each of us knows it is still in many ways a mystery to science. We have a lot of work to do before we can claim to have cracked it.

My aim in this book is to explore this challenge and suggest a framework we can use to reach a deeper understanding of the mind in terms that make sense to scientists. I see the work as a contribution to psychophysics – my term of art for the future field at the interface of psychology and physics.

Physics is the fundamental science of nature. Psychology is not yet a science in the same sense. It resembles biology before the theory of evolution gave biologists a unifying framework. Neuroscientists are nibbling away at psychology, but they still don't have a convincing theory of mind. Something is wrong, but no one seems to know what.

My diagnosis of the problem is that the framing logic for the challenge needs recasting. We need to recognize the role of becoming, or of evolution in the widest sense, to the task of building the models that do the heavy lifting in science. The idea that we're faced with a conceptually static reality, along with a geometric time dimension that exists outside of us, is wrong. It's been shown to be wrong by quantum physicists. When we respond with an open mind to this fact, the project of developing a conceptual basis for psychology can be made to look much less daunting.

But the task is still a tricky one. We need to dip into some deep and difficult logic and mathematics, and we can't avoid some alarming paradoxes that can make the whole enterprise look doomed. The journey takes us over rocky ground and seems to wander all over the landscape of our best theories about our place in nature. Happily, years of grappling with all this stuff has taught me to find ways of smoothing over the bumps and bridging the nasty patches, so the gloss presented here goes easy on its readers.

THE ELEVATOR PITCH

Given the new view of logic and math, plus the new view of quantum physics and the gusher of new facts from the neurosciences, my new perspective on psychology isn't weird at all. The weirdness was already absorbed in the givens. But it is a radically different perspective from the conventional view that faced the pioneers of psychology a hundred or more years ago. It will take some getting used to.

In short, we use a logic of becoming to distinguish the big self from the little self. If the ego is the big self in being, the little self is projected into existence as a puppet avatar in a virtual reality, or a mindworld. As conscious beings, we live in a mindworld movie. Each brief self is timestamped as it goes from being to existence, and our lived reality is a strange loop that turns and grows in time.

When it's cut this short, the new view doesn't make much sense. Any pitch short enough to deliver in an elevator ride is likely to be just as unintelligible. But if we make it a tad longer (imagine a fast talker in a slow elevator in a tall skyscraper), it might make more sense. Here goes.

When we let logic unfold into mathematical set theory, we find that a dynamic view of truth in a formalized theory of becoming looks rather interesting. When we then apply this logic to review time in physics, we find that it takes on a new character that's attractive from a psychological point of view. When we then see how the dynamic view of time unfolds in quantum theory, we find we have a promising new way to get a grip on the notorious paradoxes that make quantum physics seem impossible to understand.

This three-step development is all by way of introduction to the deeper view that follows – though it takes us halfway through the book to get that far. The science of mind we aim to glimpse is based on lots of experimental work in biology and on recent neuroscientific studies of mammalian brains, so we need to review all that next, as well as the wider debates that frame any such work. Then, in an easy application of the key results so far, the new view pops out with startling speed and clarity as a physical insight that pans out to a universal perspective of breathtaking scope.

A deep breath later, the consolidation of that outcome in logic and philosophy is reassuringly smooth. If we see reality as a movement from being to existence, we can assign mental and mathematical ideas generally to a virtual realm of being and reserve existence for the more defined and limited realm of things in spacetime. Then we can distinguish the ego in being from its avatar in existence and depict the temporal flow from ego to avatar as the frame-by-frame realization of a mindworld movie. After all that, we can regard our social life as a multiuser online game. That's the story.

GRAPPLING WITH PARADOX

It took me decades to think through the core idea, then years to sort out the rest. To find ways to bridge the yawning gaps in my early drafts of a narrative, I took some difficult detours through philosophy, logic, and mathematics.

My academic home port for embarking on the voyage was Oxford. The scientific need to work out a deeper theory of mind dawned on me in 1970 as an undergraduate reading physics who was struggling to get his head around the deeper issues stirred up by relativity and quantum theory. Then, at sea as a postgrad researcher, I navigated into philosophy, logic, scientific method, mathematical logic, foundations of math, and the philosophy of language.

A breakthrough of sorts animated me over the summer of 1974 in Berlin. I drafted a book in 1975 to sketch out my idea, which I took from handwritten notes to a bound typescript, but it didn't make much sense. There was obviously still a mountain of work ahead of me.

The problem was paradox. Classical logic, the logic we use to build computers and the internet, is intolerant of anything that even flirts with paradox and contradiction. Everything must remain rigorously consistent. This is clearly a virtue for any practical endeavor, and no sane scientists would wish to disavow a commitment to consistency. But it makes building deep theoretical foundations quite tricky.

The strategy I chose in Berlin was to stare down the contradictions that emerged in ambitious applications of logic. This would be like the two superpowers coldly staring down their political differences over the Berlin Wall. The paradoxes were deep and wide, but they weren't overwhelming.

Back in Oxford, I continued my studies, then worked in London until 1987, when I moved to Germany. There I did editorial deskwork on academic studies in math, physics, and computer science for a decade.

In the early years of the new millennium, still in Germany and working in software development, I took part in a series of conferences on the latest developments in neuroscience.

Early inklings of a new science of mind were appearing on the strength of an impressive flood of new experimental work. Theoretical physics was also looking better. Relativistic and quantum physics had become the firm basis for all the sciences and inspired new models for cosmology and particle physics. The pieces were in place to get the job done.

THIS BOOK

The time was ripe to publish or perish. I published my best essays from the previous decade in my 2009 book *Mindworlds*, retired from software development, wrote a few more books, and returned to England in 2013. A few further distractions later, I can now offer this book to the world.

The great challenge for a project like this is to identify the intended audience and then to pitch the exposition at a level that supports and respects that audience. Even expert readers deserve explanations and references that suffice to locate and define the key ideas and innovations within a familiar frame. Readers who are new to the issues raised here are especially entitled to a full and fair presentation of what for experts may seem like elementary points that surround the main novelties. The challenge is to balance the wants and needs of all such readers without being boring.

In the end, I chose to compose a light and fairly readable main text followed by relatively technical notes and references at the end for scholars. This approach has sound precedents among expositions of novel ideas in science, especially ones that seem too unconventional or controversial to be squeezed into the straitjackets of peer-reviewed journals or specialist academic monographs. I want to reach a wider audience, and this seems the best way to do so.

The book has six fact-laden chapters. A bold teacher could recycle the material as a resource for a one-semester course aimed at STEM students who want to think outside the box. But my ambition is for the book to attract readers from many different backgrounds. If it failed to excite readers outside the academic community, I'd conclude that my efforts to make it readable, lively, and provocative had been wasted.

To whet the appetite of people who like to read the menu before consuming what may be junk in books like this, here's a quick overview of the six chapters.

The first chapter, *Being*, reviews the relevant history behind the search for a scientific theory of mind and introduces the tools of formal logic and set theory we need to overcome the obstacles facing that science.

The second chapter, *Time*, applies the new view of logic to the classical conception of time, as worked out by physicists and mathematicians, and explains in more detail how time works in a theory of mind.

The third chapter, *States*, introduces quantum theory, with the aim of showing how the new logic of time enables us to avoid the air of paradox surrounding it, and hints at how the quantum ideas can help us in a theory of mind.

The fourth chapter, *Life*, starts with a review of how life on Earth has grown in modern scientific terms and continues by describing the brain and the tools we use to explore it.

The fifth chapter, *Minds*, outlines a new way to understand how the brain supports the mind and proposes a hypothesis, rooted in modern logic and physics, to explain the temporal nature of conscious experience.

The sixth chapter, *Worlds*, introduces worlds of consciousness as mindworlds and then presents and explains nine laws of psychophysics to summarize the book's key message.

Between the fascinating details (some of them arcane), the main argument should be easy to follow. Though my own story is irrelevant to the case made here, I've included a few minor biographical comments where the extra facts seemed helpful. To make for easy reading, I've suppressed footnote markers in the main text. The notes and references are for specialists, and most readers will have no problem ignoring them. I've also suppressed URL and doi data in the references – motivated readers can use the cited text to locate resources online with an intelligent search app.

This is not a scientific monograph in the traditional sense. But it is intended to introduce a perspective that makes a real contribution to serious science. As I said, I've made an effort to keep the journey interesting for readers who not only share my ambition to reach the destination but also want to enjoy the ride.

England, 2025

KANT'S HAIKU

Thoughts without content
are empty, and sense inputs
without concepts blind.

BEING

Once upon a time, Germany was a land of thinkers and poets. The kingdom of Prussia and the patchwork of statelets left over from the Holy Roman Empire fostered gifted musicians, brilliant mathematicians, talented scientists, and the greatest harvest of philosophers since ancient Greece.

Foremost among the philosophers was Immanuel Kant, a scholar versed in physics and math who first suggested that galaxies were “island universes” and who made a monumental contribution to the theory of mind. His great insight was to grasp that we never experience the world directly but always through what he called the “lenses” of categories within the “æsthetic” of space and time. We have no immediate view of the real world and see only the phenomenal world through these lenses. Kant’s historic contribution was to argue that any rational being, anywhere in the universe, is constrained to apprehend reality through a set of categories.

Pre-eminent among the philosophers who followed Kant was Georg W.F. Hegel, who conceived an amazing synthesis of that theory of mind with a dialectical history of everything that put its stamp on Western philosophy for the next hundred years. Hegel sought to outdo Kant by dismissing the idea of a real world beyond the categories. He found ways to fit every aspect of human life into a dialectical framework that exposed it to rational reconstruction within a nexus of logical relationships. In doing so, he revealed the limits of his own logic. His early disciples included Karl Marx, whose revolutionary ideas transformed politics in the twentieth century.

The philosopher whose career heralded the eclipse of the German love of genius in the apocalyptic demise of the Third Reich was Martin Heidegger. His main achievement was to resurrect the ancient Greek concept of being in a way that invited a new approach to building a theory of mind. The French existentialist movement in philosophy and psychology arose from Heidegger's ideas.

Germany's blaze of brilliance died in the ashes of the Third Reich. Philosophy in the distinctive German manner lost its spark, and physics took over as the more promising discipline for advancing our basic knowledge about reality. In the early twentieth century, physicists developed a new understanding of space and time and discovered a new world of ideas in quantum mechanics to replace the classical understanding of the physical world that had persisted in its deepest essentials for two thousand years. Thinkers who pursued the decay of Hegelianism into Marxism offered only stale dogma. In the new world, atomic bombs made physicists more important.

Today, with computers, artificial intelligence, and robot lab technology, the industrialization of science has grown from physics and chemistry to include biology and medical science, where its impact has been transformational. Science has gone global. Its next frontier challenge is psychology.

MIND THE GAP

My main purpose in this book is to trace out the implications of a radical perspective in logic for the project of building a scientific theory of mind. This lets us formulate the central paradox regarding the relation between mind and matter in a way that makes scientific sense. By exploring the interface between the embryonic science of psychology and the mature science of physics, as well as the firehose of facts emerging from work in the neurosciences, we can lay the foundations for a viable science of psychophysics.

Kant made a start. He didn't get far with it, because there was so little scientific work to build on, but he said his start marked a Copernican revolution in how we understand the relation between the mind and the world. Recall that Nicolaus Copernicus was the Renaissance man who replaced classical Ptolemaic cosmology, which put the Earth at the fixed center of the universe, with the heliocentric view that Earth and the other planets orbit the Sun. Kant said we see reality through a veil of phenomena, and the categories we apply to understand phenomena are as subjective as the idea of a fixed Earth. Reality is stranger than we can grasp.

Modern physics presupposes observers. We are observers. Our minds give us a perspective on the reality that surrounds and sustains us. Our subjective minds reflect a spatiotemporal world of physical phenomena. In modern physics, the deeper reality is a labyrinth of mathematical forms.

Yet minds are somehow spatial. Most psychologists take our concepts of space and time as subjective in the Kantian sense, but they also accept – naively, perhaps – that in reality we also have definite locations in space and time.

Here we're faced with some awkward choices. Mathematical objects are separate from each other, yet they lack location in space and time. Perhaps our minds (or souls – the distinction between them is far from clear) are similar. Perhaps mental phenomena generally have a being or existence like that of mathematical objects. Minds are often compared to software running on the brain, and we all agree that software is akin to mathematics – algorithmic programs process information in the same way that arithmetic calculations process numbers.

Numbers or information can claim to be eternal residents in Plato's heaven, whereas grubby calculations on paper or buggy code in a computer register are as subject to change and decay as anything in the physical world.

Space and time are formal concepts we use to order reality and make sense of the world. Without a geometry for space and time, we couldn't distinguish physical objects from each other or make sense of their changing. Even today, almost all physicists say space and time are as real as it gets.

Following the Copernican revolution, physics in our sense began when Galileo Galilei and Isaac Newton proposed laws in mathematical form to account for exact measurements of physical phenomena. While the genial polymath Gottfried Wilhelm Leibniz proposed a relational view of space and time, his scientific nemesis Newton said absolute space and time exist in physical reality as the imperturbable foundation of the laws of mechanics and the inverse-square law of gravitation. Newton's view prevailed for centuries, until Albert Einstein showed us how to relativize these absolutes.

The conventional view in physics is that the observer is a perspectival point located in space and time. When Kant explored how the observer's mind relates to space and time, and then Hegel and Heidegger explored the idea further, they gave us the stub of a line we can continue. We can say the spatiotemporal realm of things exists within a wider realm of being that includes mind and mathematics.

In the philosophy we get from this stub, human beings are people with minds, and minds have being, whereas physical bodies exist. Being is somehow distinct from existence.

Let's push on and see how far this goes. We can assert that minds have moral and metaphysical attributes that go beyond anything we might casually grant to tables and chairs. Maybe souls have the moral and metaphysical attributes, and minds come along for the ride, but then we'd need a story of souls, for example that minds are the user interfaces for souls, and self-consciousness extends to minds but not to souls.

We shall forget about souls and so on and stick to minds. But consciousness is something we can't forget about.

LIVING WITH CONSCIOUSNESS

In recent decades, consciousness has become the main battleground for a theory of mind. It's what distinguishes human beings with minds from humanoid zombies or meat machines. On this view, aired by the philosopher David J. Chalmers, to have consciousness is to have an inner life or a subjective experience of the world. Chalmers gained academic fame as the young man who sang "the zombie blues" with rock-star zest to promote his claim that the hard problem in advancing from the neuroscience of cognitive processing in the brain to a scientific theory of mind for psychology is to build a theory of consciousness.

Minds are closely related to consciousness in the sense that they provide personal windows or theaters of consciousness. Explaining consciousness is the key step in explaining minds. Both are big, baggy ideas, perhaps too big for easy scientific assimilation. A scientist can reasonably focus more narrowly on states of mind. We can admit that states of mind exist but insist nonetheless that minds are too nebulous for existence. This may seem inconsequential, like mere wordplay, but we're working toward a powerful mathematical framework here, so let's not abandon our approach until the fuller picture begins to take shape and the motivation for this distinction between being and existence emerges.

Zooming out again to physics, we might also ask whether space and time exist. Many things exist within space and time, certainly, but we can argue in Kantian fashion that space and time must be prior to phenomenal existence. For the physical objects we perceive, existence implies spatiotemporal location, which space and time themselves lack.

Does existence itself exist? The totality of existence, which we can identify with the universe, has some kind of being, but again this is too vague for existence. Even within cosmology, the term "universe" is hard to pin down. Does it denote the observable universe lying within the spherical event horizon around Earth that marks the cosmic dawn of the first light, which is now stretched out as the cosmic microwave background? Or does it extend to the entire Big Bang bubble that includes vast stretches of spacetime forever cut off from us? Or does it extend to the oceanic inflationary multiverse – the cosmic bubble bath – or even to a quantum multiverse with preposterously many infinitesimally different parallel worlds? Existence must end somewhere.

The existing entities corresponding to the universe can be said to be worlds of some kind that we can define. Any more generous policy invites confusion. We're welcome to say that plenty of worlds exist, on the understanding that worlds are limited and specific entities of some kind.

All this is by way of introducing the concepts of being and existence and of motivating a distinction between them. Being, we may say, is half of existence, and existence is definite being. A logical story of becoming will motivate a dynamic view of their contrast, but first we need to set the stage.

Things exist. Physics is the science of spelling out more exactly what this means and what laws or regularities govern these things as they interact with each other. Psychology is the study of how we, as subjects with minds, perceive things and discern truths about them.

Physics is as old as science itself, and today its exactitude and reliability are exemplary for scientists. Following the twin revolutions of relativity theory and quantum mechanics in the twentieth century, we now have two standard models. The particle physics model describes matter in terms of vast numbers of tiny and typically transient particles – which are equally wave phenomena – subject to quantum field theory. The cosmology model covers our Big Bang universe, with its population of galaxies and black holes located in curved spacetime. Those two models together – so long as we gloss over a few frontier issues – form a foundation that looks firm enough for all the sciences.

Psychology, by contrast, is a fledgling science with disputed data and unsteady concepts. Some say it first became scientific when the American psychologist William James published his *Principles of Psychology* in 1890, but others say the real science began much later. Between physics and psychology is biology, which first became a science when Charles Darwin gave the field an illuminating theoretical framework with his theory of evolution by natural selection, and then – a busy century later – physicists and chemists discovered and mapped the details of the mechanism of genetic inheritance via DNA molecules. Today, with the neurosciences delivering new data and new insights every day, the prospects look brighter than ever for a solidly founded science of mind, yet we still lack a physically plausible theory of consciousness.

Key to the success of physics is mathematics. Mathematical concepts support and define all the main ideas and theories of physics. More accurately, we define the play of these concepts using logic, which supports the formal grammar for math, which in turn supports the conceptual structures of physics, and by extension the other sciences too.

Mathematics seems to lack the experimental side that most people expect to find in a science. But it's fair to say that toying with math on computers can be experimental in spirit, so that contrast with the sciences is moot. Also, aesthetic criteria play a larger part in math than in most sciences, but that's not an absolute contrast. Those points aside, even skeptics call math the queen of the sciences.

We can see math as like a science of mind. Mathematical imagery is fantasy made logical. But there's more. Accepting the sovereignty of math is prerequisite for doing any real scientific work, and its sovereignty turns on the role of logic. We can say logic is the key thing we need to do math – even more so than pencil and paper or computer and connectivity. We can regard logic as setting the ground rules for the games we all play with math.

Logic is where it all begins, and here the foundation is bare enough to appear barren at first sight. A distinction between truth and falsity, which can boil down to a distinction between 0 and 1, is enough to get logic moving. All the rest amounts to rules and conventions, which make up a formal grammar for distinguishing truths from falsehoods and tracking truth using rules of inference. We say the rules are valid if they never lead from truths to falsehoods and thus preserve consistency.

Centuries ago, the metaphysics of this primal distinction preoccupied philosophers. Today, unsurprisingly, such metaphysics can seem casuistic, if not demonstrably nonsensical. We start out with the apparent contradiction that nothing is, in being, yet being isn't nothing, and we claim to resolve this antithesis by recognizing their reciprocal transition within the process of becoming. Empty being is void, which we can code as 0. We can then code being as it exists in contrast to nothing as 1. If we define what exists in contrast to what is, then we have our primal distinction.

Today, with the advent of qubits in quantum information theory, we're used to the idea that 0 and 1 – the two states of a bit in classical computing – are not as simple as they seem when we dig deeper into the physics of information. Qubits can be in superpositions of 0 and 1 and can be entangled with each other, which makes quantum logic far subtler than anything imagined before. But that's a topic for later.

Let's return briefly to traditional metaphysics. Existence – where the prefix "ex" qualifies something past or voided, the verb "is" indicates being, and the suffix makes the word a noun – suggests past being, or being overcome, tending to 0. In its place dawns a new manifestation of being, which embraces that existence and defines it as 1.

A premodern metaphysician might take this as a cue to spin out an alternating succession of flips between 0 and 1 along a dimension of becoming, which we can regard as the abstract equivalent of a timeline. The overarching purpose of this process is to elaborate on the things strung along this logical dimension in order to clarify the distinctions between them and to build up a growing pile of things with ever more clearly determined properties. The outcome of this cumulative or evolutionary development of clarity (or of "truth"), powered by repeated flipping between 0 and 1, is something that two hundred years ago – which counts as the prehistory of modern analytic philosophy – the historically-minded Hegel called a dialectic. Today, it seems to me that we can more helpfully (though only in hindsight) regard such a dialectic as a cloudy and protean vision for set theory, which is a branch of math first developed since Hegel's time.

ALL YOU NEED IS SETS

Set theory provides the logical frame within which we can reconcile physics with psychology – or at least this is my big claim in this book. I first glimpsed how to do so some fifty years ago, but it took me a few decades to clarify the story far enough to tell it with any real hope of persuading skeptics. Happily, the mathematical details, which soon get dauntingly difficult, are irrelevant for our purposes, and an outline we can describe in ordinary words will do fine. Still, it's tricky stuff that's best developed slowly, with continuing hints to relate the unfolding picture to a theory of mind.

Crudely, set theory is about the most abstract possible form of a theory of everything. Anything can be seen as a set if we squint hard enough. In the twentieth century, the American logician Willard van Orman Quine, who not only developed two distinctive systems of set theory but also introduced the idea of ontologies into modern debate, opined that anything can be modeled logically or formally in set theory, so the ultimate ontology is simply a universe of sets. That bold claim may or may not be true, but we can indeed model just about anything in math in set theory, which is a start. The rest, as even fans of the approach admit, is up for grabs.

Quine's ultimate ontology of sets may seem too extreme to be taken seriously. Oddly, it parallels Quine's good friend the psychologist B.F. Skinner's dubious idea that all mental life, in humans or in pigeons, can be reduced to stimulus–response behavior. Worse, it's reminiscent of the physicist Richard P. Feynman's joke that anyone looking for an equation for a theory of everything to print on a T-shirt could simply print "U = 0" by defining U, for unworldliness, as the sum of a series of deviations from the established laws of physics. In short, you can say what you like if you don't care about the details, but getting the details right is what science is all about, and you can't cheat by inventing a slogan.

In fact, there is hope for something like Quine's ultimate ontology. Physicists are warming to the idea that everything in physics can be reduced to qubits, or quanta of information that pop into existence as 0 or 1 whenever we try to measure them. Mapping qubits to sets is a nontrivial challenge, but given the metaphysical perspective on being and existence we've just reviewed, it looks doable.

The inspiration for set theory – the basic idea of a set – first emerged in the nineteenth century. The mathematician Georg Cantor was among the early pioneers. His bold new ideas met with strong professional resistance and frequent scorn, but he succeeded in using his unorthodox understanding of sets to develop an extraordinary new world of transfinite arithmetic. We needn't go deep into his transfinite paradise here, but his story is worth remembering by those who fear that all these ideas are too wild, because it reminds us that set theory has been thoroughly stress-tested. It's the application of the ideas in physics and psychology that deserves a more critical stance. My main challenge in this book is to make such application seem interesting and helpful.

Let's outline plainly what sets are. A set is a thing defined by its members. A set has members and is itself a member of further sets. The membership relation between a set and its members is the intended interpretation of the only predicate we need to say anything we wish to say about sets. As a starter, we can define identity for sets by appeal to their membership relations: If what appear to be two sets denoted by distinct expressions A and B turn out to have the same members, then A and B denote one and the same set.

Within pure math, we can go further, because the logical relations between mathematical entities like numbers define them. We can model all of pure math in an empty universe where the only sets are pure sets built up from the empty set (let's call it 0) by taking into our ontology set 0, then the set whose only member is 0 (call it $\{0\}$), then the set whose two members are 0 and the set whose only member is 0 (that is, the set $\{0, \{0\}\}$), and so on.

These simple pure sets let us model the natural numbers rather naturally. Several ways to do so were proposed, but by common consent the best one is due to the brilliant polymath John von Neumann, who not only did creative work in set theory and quantum mechanics but also designed the default architecture for almost all modern computers, used his early computer at Princeton to help design hydrogen bombs, and fathered game theory.

The ingenious way von Neumann defined the numbers is suggestive of how the whole project of founding math in logic can be pursued, so let's take a brief look at it. We call the sets he used to define numbers the von Neumann ordinals. In transfinite arithmetic, we distinguish ordinal numbers, which we use for counting, from cardinal numbers, which we use to define size for infinite sets, but here and now we need only consider the finite natural numbers.

He used a process of induction by starting with zero and then defining for each number n its successor (which for 0 is 1, for 1 is 2, and so on). Iterating the application of this definition now takes us step by step into countable infinity. He took 0 as the empty set, as usual, but then defined the set standing for the successor $S(n)$ of any number n as follows: Assuming that n maps to set x, we say the set for $S(n)$ is the union of x with the set $\{x\}$ that has x as its sole member. As in everyday Venn diagrams, the union of sets A and B is the set containing the members of A, the members of B, and the members of both A and B.

This modeling of numbers gives us the intuitive picture that each number *n* is mapped to the set of all the sets for numbers smaller than *n*. In this mapping, 1 is the set of 0; 2 is the set of 0 and 1; 3 is the set of 0, 1, and 2; and so on. Conveniently, any number *m* is less than any other number *n* whenever the set mapped to *m* is a member of the set mapped to *n*. Also, the cardinality of each number *n* is *n* itself, so the finite ordinals and finite cardinals coincide.

Given sets for the natural numbers, the way is clear to model the rest of math in set theory. We can model the basic numerical operations of addition and multiplication as suitable operations on sets; we can define sets for negative integers, rational numbers, real numbers, and complex numbers; we can define functions over sets for more advanced operations, such as infinite sums and products, taking limits; and so on. Doing all this takes lots of patience and plenty of time, and the work took decades to deliver a stable theory.

The first mathematician to model large parts of math in set theory was Gottlob Frege, who is now lauded as the founder of modern logic and a founding father of the perspective that dominates Anglo-American analytic philosophy.

Frege thought of his set theory as logic and conceived his enterprise as one of reducing arithmetic to logic in line with Kant's view of their relationship. Frege saw his own formalism for set theory as marking a hard reboot in logic to replace the cloudy and nonsensical word salad he discerned in Hegelian dialectics. But to his consternation, his set theory gave rise to a logical contradiction, which soon led to a community-wide effort of reconstruction that resulted decades later in a better theory. Ironically, the new and improved theory in my view resembles a Hegelian dialectic, one transformed into something sharp and clear, shorn of redundant words and reduced to a reliable formal tower of abstractions.

The story of the reconstruction of set theory is worth telling here mainly because it introduces a paradox that hit me like a thunderbolt fifty years ago and still serves as the central insight behind my physical theory of mind.

In brief, Frege said that any and every definable collection of things, be they sets or numbers or tables and chairs, is a set. Almost immediately, the Cambridge mathematician Bertrand A.W. Russell pointed out that the set of all sets that are not members of themselves could not itself be a set, or we get the contradiction that the set is a member of itself if and only if it's not a member of itself. This contradiction was fundamental and definitive for the fortunes of set theory. Russell and other mathematicians spent the first few decades of the twentieth century trying to sort out the mess.

At about the same time, Cantor also faced a contradiction. His signature achievement in math was to discover that there is more than one order of infinity. The infinity of natural numbers, countable infinity, is only the start. If we call the set of natural numbers N and define the power set of N as the set of all possible subsets of N (which for infinite sets is a more ambitious and contentious idea than it might seem), Cantor discovered a clear sense in which the cardinality of the power set of N is uncountably infinite.

A brief drilldown here may look like mathematical overkill, but the argument Cantor used has been reapplied in so many ways since, including in consciousness studies, that it's worth a paragraph or two. Cantor used a diagonal argument to prove that the power set of N is strictly bigger than N. Any infinite list of numbers is countable, so any infinite set of numbers that can be listed in such a way that any member of the set will appear eventually in the list – such as the set of rational numbers, which are numbers that can be written as fractions – has the same cardinality as N. Conversely, any set with more members than we can put into a list is uncountable.

Cantor used his diagonal argument to prove that the set of all real numbers between 0 and 1 is uncountable, where a real number is defined by any infinite string of digits. He already knew that the set of all rational numbers between 0 and 1 is countable, so his argument showed that there were more reals than rationals between 0 and 1, and indeed more reals between 0 and 1 than there are rationals on an infinite line.

Let's give the proof. Real numbers can be written as infinite fractions, but it doesn't matter which base you use, such as decimal or binary, because you can always translate between bases. In binary, the reals between 0 and 1 can all be coded as infinite bit strings, with each bit as 0 or 1.

Imagine trying to write out the reals between 0 and 1 in a list. Let the rows in the list be strings combining bits in any way you like to represent reals in the interval. The result is an infinite 2D array of digits. Draw a diagonal line from top left to bottom right, so as to cross the nth row at the nth digit, and read off the digits on the line. Now write a new row with a different digit at each nth place than the digit on the nth row at that place. The new row will be an infinite bit string that never appears in the list, however far down it you scroll. The list fails to include all the real numbers between 0 and 1. This is Cantor's famous diagonal argument.

Cantor generalized his proof to the case where the power set was defined for an uncountable set. For any infinite set x, whether it's countable or uncountable, the power set of x has a higher cardinality than x. The result is a transfinite hierarchy of orders of infinity – also known as Cantor's paradise.

That was probably more math than you expected to find in an informal introduction to psychophysics, but the point is that a diagonal argument plays a key role in the recent debate about consciousness. Shortly before the turn of the century, Chalmers used a version of the diagonal argument in his book *The Conscious Mind*, and his claim has resonated among the philosophical members of the neuroscience community ever since. In short, he maintained that no amount of neuroscience can ever explain the first-person experience of consciousness because however you slice the science it's still a third-person activity that misses what it's like to be me. We shall revisit this issue in depth later, so this reference is just a marker.

BUILDING A CLASS HIERARCHY

The contradiction Cantor faced was triggered by asking what was the cardinality of the set of all cardinals. Cantor had just proved that it's higher than anything in that set! He concluded that there is no set of all cardinals. Frege should have seen a similar problem coming from his own definitions. There were other, similar paradoxes in set theory, all stemming from statements involving both universals and self-reference. Something deep was wrong there.

Eventually, what emerged in axiomatic set theory was what we call the cumulative hierarchy of sets. The guiding idea was that each new set could only have previously existing sets as members. Self-membership was forbidden by the formalism. The cumulative hierarchy was built up methodically from a minimal start, layer by layer, and only sets built up in this way were accepted as existing.

The mathematician who first defined this structure was Ernst Zermelo. His hierarchy was later extended into a bigger structure, one big enough to encompass all of classical math, by Abraham A. Fraenkel, and so the resulting theory is called Zermelo–Fraenkel (ZF) set theory. It has since been accepted as the standard foundation of mathematics.

For the purpose of building that foundation, all these sets can be pure sets, all based ultimately on 0, with nothing else admitted into the ontology. In von Neumann's formulation, all these sets are ranked as members of successive indexed subuniverses of sets that are each in turn members of the ultimate universe of sets. That ultimate universe can't itself be a set – if it were, then by definition it would be a member of itself, and we'd have the same old contradiction. Instead, each ranked subuniverse is the set of all sets that can be formed from sets of lower ranks in the hierarchy.

These subuniverses are power sets. A running ordinal index ranks all the power sets into a cumulative hierarchy of pure sets that marches inexorably onward into Cantor's paradise. The higher power sets have cardinalities that soon overwhelm any formal system we can devise.

In my opinion, the most intuitive way to describe how the cumulative hierarchy avoids the paradoxes is to distinguish sets from classes. Sets have two sides, corresponding to the image that they look down on their members but look up to the sets of which they're a member. Any set is the class of its members, which lie below it, and a member of various classes that hover above it. Any set is a member from above and a class from below. And we must start from the bottom.

Confusingly, in their milestone trilogy *Principia Mathematica*, Russell and his collaborator A.N. Whitehead called all sets classes. Fortunately, other theorists, including von Neumann, Paul Bernays, and Kurt Gödel, distinguished sets from classes. In his survey volume *Set Theory and Its Logic*, Quine detailed different axiomatizations of set theory and called their theory NBG to distinguish it from ZF set theory. It turns out that NBG is formally exactly like ZF except for this distinction between sets and classes, so for simplicity we just talk about ZF as the foundation for math.

Given the story of sets and classes, we can at last spell out the distinction between being and existence. Sets exist, but classes need only have being. Sets have a topside as well as an inside, and classes have an inside. Sets are things that together form an ontology, but classes aren't and don't. Think of the universe of sets (which we usually call V) as a mathematical heaven, where classes form the population of beings in that heaven. Among them, proper classes are beings that don't yet exist. All sets are classes too, but no sets are proper classes. Proper classes are the hard cases that could lead to paradoxes if we let them exist.

A quibble about the empty set 0 is natural at this stage. If a set is made up of its members, how can a set have nothing in it? The empty set has nothing inside, which lets us regard it as having no inside. By analogy with proper classes, we can call it a proper member. Zooming out to the universe V again, the cumulative hierarchy, strung along its logical dimension in ranks, is topped and tailed by V and 0, respectively. Nothing below, nothing above; an abstract ontology of sets is complete and self-sufficient, so long as we adopt Quine's line that all you need is sets.

Reasonable people, when first introduced to Quine's line, often complain that most things aren't sets. Pure set theory is a glass bead game that may amuse mathematicians, but it has no relevance to the everyday lives of physicists and ordinary people except in the sense that it provides a reassuring logical backstop for the math they use.

But I think that conclusion would be premature. We can map nested structures into set theory in arbitrary ways, so long as the logic is preserved, and transfinite set theory has the ontological depth to swallow any amount of complexity in the relations between things. Given a suitable mapping, the idea that sets are collections of things need not be translated. We just have a logical network with a map.

Looking ahead, an artificial neural network is essentially a logical network in which the layers of neurons are logically much like ranks in the cumulative hierarchy. Each layer of neurons in a neural network maps to sets of the appropriate rank. We can read out from a network trained to, say, classify images a logical structure that gives us a set for each image. If our minds can be modeled by such neural networks, this can automate for us the process of building up a Quinean ontology for our inventory of images.

We can leave such exercises to artificial intelligence geeks. Our claim here is only that the set hierarchy is at least a good formal metaphor for logical structure and hence a formalism that can have deep explanatory power. That depth arises, as I hope to make clear, from the universal paradox, which finds a vivid analog in the theory of mind.

ENTER THE OUROBOROS

Ludwig Wittgenstein began his academic career as an intense young student who studied under Russell and dazzled British academics with his genius. Years later, as a tenured Cambridge professor, he recorded his thoughts on numerous scraps of paper that were edited after his death by his disciples. On one such scrap, he wrote an unusually cryptic remark that was included in a volume his disciples edited and translated on the foundations of math. In the remark, Wittgenstein bemoaned the "Janus-faced" nature of mathematical formalisms built by eager but philosophically naïve formalists who imagined their work might make the foundation more secure.

When I read this cryptic remark, I immediately interpreted it as commenting on how the paradoxes of set theory revealed a universe of sets looped around on itself like an ouroboros – the ancient Greek name for a symbol depicting a snake biting its own tail. My interpretation revealed more about me than about Wittgenstein, but there it was: The loop was lurking in the math already, whether he liked it or not.

For Wittgenstein in his Cambridge study, this was just a throwaway remark, but for me fifty years ago it was a vivid reminder of the revelation I'd enjoyed only months earlier while contemplating the dialectics of set theory. I was blasé about Wittgenstein's opinion – mathematicians tended to dismiss his work as lacking method – but I was delighted to find this resonance with my own view that the paradoxes weren't just technical errors to be smoothed away.

My version of the story went thus: By letting us join the foggy bottom of 0 with the cloudy top of V into a loop like an ouroboros with a "Janus-faced" bite zone we might dub V|0, set theory seemed to neutralize the kind of rhetorical contradiction that a century earlier made logicians despair and gave dialectics a bad name. In my new vision, the hierarchy stretches the loop out straight along its logical dimension and veils the ends in a foggy cloud of unknowing, thus transforming a source of contradictions – of short circuits in the logical networks implemented in the universe – into a harmless and even useful background paradox.

Despite this apparently benign transformation, the work needed to iron out the wrinkles and apply the laundered result to practical issues in science has taken me decades. The dark specter of the ouroboros still lurks behind the theory I've worked out, but I can reassure you that in this book it does nothing more evil than power the thunderbolt that makes minds come alive.

SUMMARY

Our brief excursion into mathematical mysticism has taken us into an abyss of abstraction. The abyss is deep enough to form a shared foundation for physics and psychology, as I hope to show. The ontogenesis of the universe at the dawn of time is almost begging to repose on the sort of metaphysics that my story of 0 and V represents. Similarly, an account of the dawn of a mind in the initially blank subjectivity of a new biological organism fairly begs to be based on an architecture of neural circuitry that starts at 0 and grows to comprehend what I call mindworlds in consciousness.

The contrast of being and existence and its articulation in the dialectic of 0 and 1 along a logical dimension of becoming analogous to a timeline is as challenging as it gets in this essay. We've made a start, and the rest is leveraging the V|0 paradox to trace the emergence of time, states, life, minds, and worlds.

EINSTEIN’S HAIKU

Energy is mass,
the speed of light is constant,
and spacetime is curved.

TIME

The classical concept of time is now inseparable from Albert Einstein’s special and general theories of relativity. The special theory has led us to define spacetime as an indivisible four-dimensional continuum, and the general theory accounts for gravity as the effect of curved spacetime. Flushed with these successes, Einstein was tempted to think he’d been granted a glimpse of the eternal reality behind what Kant would call the veil of appearances.

Einstein was aided and abetted in this thought by his good friend Kurt Gödel. The physicist and the mathematician met every day during their years as distinguished senior fellows in Princeton. Gödel had gained fame as a young man by proving that arithmetic and similar theories were incomplete, by which he meant there were sentences of their formal languages that could be neither proved nor disproved within those theories. He followed this incompleteness theorem with a second one proving that the consistency of arithmetic and similar theories could not be proved within those theories.

Gödel’s two incompleteness theorems caused a revolution in the philosophy of math and logic and prompted a wave of further work in both fields. His theorems, together with the mindset he adopted to prove them, gave Gödel a firm belief that mathematical truths are eternal.

At Princeton, Gödel found a new solution of Einstein’s cosmological field equation. For those who don’t know it, this is the four-dimensional tensor formula that summarizes the coupled differential equations Einstein wrote in 1915 to relate the curvature of 4D spacetime with the distribution of mass and energy in the universe. The equation is famously hard to solve, but it enabled those who found solutions to predict an expanding universe, the Big Bang, and black holes.

Gödel's new solution permitted the existence of time loops in a rotating universe. As Einstein told him, this scenario was highly unphysical, but the point was that the math allowed it. This surely said something about time.

Both men agreed that reality is an eternal block, so that our human perception of the flow of time is a mere psychological limitation on the way we perceive reality. Both of them had read Kant's works when younger, and both regarded their discoveries about the block universe as providing an almost mystic glimpse beyond the veil of phenomena.

Their Olympian view of reality contrasts with a widespread modern misunderstanding of science and expert opinion. We can agree that a human perspective on nature – the view from a personal mind – is limited by a time horizon which is often so close to the lived moment that it's impossible to ignore. The misunderstanding is to become so skeptical about science by emphasizing its controversies and its uncertainty as to say we can know facts about the past but can barely more than guess claims about the future. In climate science, such resistance to prediction can have apocalyptic consequences.

Philosophers of science have colluded in encouraging this skepticism. Karl Popper maintained that scientific theories are hypotheses or conjectures that are in effect tested every time they are applied. A prediction based on a theory, even one as compelling as general relativity, is always a hostage to fortune. If the facts turn out not to be as predicted and no explanation of the discrepancy pops up, we must conclude that the theory has been refuted.

We can sympathize with this view as an improvement on the widespread earlier belief that scientists collected facts and that even their theories became knowledge once confirmed by experience. In that earlier view, physics had achieved certainty with Newtonian mechanics, and classical mathematics was approaching complete understanding of the eternal truths of arithmetic and geometry. At the end of the nineteenth century, that confident view was common.

Today, we can firmly assert that both the confident and the skeptical views are partly right and partly wrong. The human predicament is that we must believe something, on pain of extinction. Either we believe the best that science can offer or we fall back on unreliable common sense, factoids culled from social media, and favorite crackpot theories. Our only viable option is to accept good science.

Our most basic beliefs about reality are so bound up with deeply embedded theories – like those of numbers and shapes in basic math, of relativity in physics, or of evolution in biology – that we cannot abandon them without sacrificing much of what we need to make sense of the world. Given the way such deep parts of science hang together, we're bound in logic to find it almost inconceivable that we could be fundamentally wrong about them. If there were deep problems there, we'd have to keep right on acting as if we believed in them until something clearly better came along.

This point about belief in science is easy to misunderstand in a way that would make building a theory of psychophysics seem meaningless, so it's worth a few more words. Work in recent decades, following up on Wittgenstein's remarks on the meaning of what we say in our natural languages, shows that the meaning of our words is not fixed forever – and neither is it so fluid that words are simply up for grabs by any fool who feels like changing them. The better view is that meanings evolve in a language community to reflect usage and precedent in ways that for the central words in our vocabulary are highly resistant to change. For the terminology of math and logic, change requires more or less unanimous professional buy-in, which can take generations to achieve. For more colloquial words, meanings can come and go like fashion trends. The word "time" has a meaning that carries structural weight in science and hence is resistant to change – as early difficulties accepting Einstein's theories attest.

A related misunderstanding is that belief in science is like belief in a religion, where faith despite the facts may be asked of believers. The practice of science requires no faith in this sense; indeed doing science usually requires robust acceptance of hard criticism and a readiness to test even the most obvious claims before accepting them. In consequence, we can believe scientific claims whenever we have good grounds to believe that they've already been tested almost to destruction within the scientific community.

The classical account of time, which today means Einstein's relativistic account, is one of the best and most developed models of physical reality we have at our disposal to help us build a psychological theory of the experience of time. Let's take the time to spell out a little more fully why Einstein and Gödel were so ready to sign off on it.

CLASSICAL SPACETIME

The story begins with the ancient Greek geometer Euclid. Euclidean geometry, the theory of straight lines, right angles, and so on, shaped Western thought for two thousand years, until the early nineteenth century, when Carl Friedrich Gauss showed that non-Euclidean geometries were possible, and his student Bernhard Riemann later spelled out how we could define a non-Euclidean metric from within, even for a space with more than three dimensions. Kant had simply presumed that Euclidean geometry was true for phenomenal space, but Gauss and Riemann opened Pandora's box.

The next strand of the story starts with the experimental discovery of the laws of electricity and magnetism in the nineteenth century. James Clerk Maxwell put these laws into a tidy mathematical form in his theory of electromagnetism, which enabled him to predict that light is electromagnetic radiation propagating at a fixed speed and that a spectrum of radiation with longer and shorter wavelengths than light lay waiting to be revealed. A century later, in his famous lectures on physics for Apollo project scientists, Richard Feynman said the advance that Maxwell's equations express would still loom large ten thousand years from now.

The constancy of the speed of light was a serious problem in Newtonian mechanics, which inherited Galilean relativity and its intuitively natural account of how the measured speed of anything depended on the relative speed of the source and the observer (speeding cars on opposite lanes of the freeway collide at the sum of their speeds). Hendrick Lorentz solved the formal problem of reconciling Newton's mechanics and Maxwell's electrodynamics with equations that seemed highly counterintuitive. The Lorentz transformation invoked weird distortions of space and time to accommodate the symmetry that Maxwell's equations displayed.

The rest of the story is well known. In 1905, Einstein made sure to conform to the Lorentz symmetry in his special theory of relativity. In 1915, he used Riemann's methods in geometry to formulate his general theory. In both cases, Einstein used bold and brilliant intuition to reconcile the physical facts with the available formal tools in such an elegant way that it would be very difficult for us to spurn the theories.

Relativistic time is one dimension among four. The relations between them are set by the Lorentz equations and scaled by the speed of light, c. Einstein's friend Hermann Minkowski defined a 4D geometry for relativistic spacetime with the key feature that its Pythagorean signature includes a minus sign for the time component – which needs explaining.

The Pythagorean theorem for a triangle in two dimensions equates the square on the hypotenuse to the sum of the squares on the other two sides. In three dimensions, it equates the square of the length of an interval (a line between two points) to the sum of the squares of the projections of the interval onto the three coordinate axes (often called x, y, and z). In Minkowski spacetime, it equates the square of the length of an interval to the sum of the squares of its projections onto the three spatial axes minus the square of its projection onto the time axis (often labeled ct, where c is the speed of light in a vacuum, which here both serves as a scale factor and restores dimensional consistency when we're using conventional units for space and time).

This key difference has an odd consequence. Imagine flying the shortest route from event A to event B in a spaceship. Given the fixed spacetime interval from A to B, this lets you go at the lowest speed to get there on time. Any other route would involve accelerating to go faster and then decelerating again, wasting energy. It follows from the Lorentz equations that all that extra action would also dilate the time you feel – also called your proper time in this context – and enable you to get to B feeling younger than if you went the direct way. You'd be right to feel that. Your body clock did indeed run more slowly during the journey. Strange to say, the shortest route maximizes the amount you age during your journey. That's the price of saving energy!

It probably goes without saying that this effect is utterly negligible at the speeds we fly on Earth, and we'd have to go at near light speed to really feel the difference. But the effect is extreme for anything traveling at light speed, such as a light beam made of the quanta we call photons. The proper time a photon feels flying from any A to B is precisely zero.

As Einstein said, time stands still for a photon. For any massive body, the speed of light might as well be infinite, because the body needs infinite energy to accelerate that far; whereas light, already going at speed *c*, takes zero proper time to get anywhere at all, as if its speed is infinite too. Another way to say this is that photons follow a path in spacetime that relativists call a null infinity.

This behavior of light rays may seem merely bizarre at first sight, but it has a major consequence for the theory of mind developed here. We go into all this later, but since the brain works with electrical impulses that generate pulsating fields, which consist of photons, it should come as no surprise that time standing still for photons has some impact on a scientific theory of mind.

Consciousness is temporal. Its apparent infinity is bounded by photons in spacetime. Any experience of eternity we enjoy is the more modest experience of photonic null infinities, or so I shall suggest. We are "thrown" (as Heidegger put it) into time. Because time is only one of four interrelated dimensions, we're limited in space too. Our minds may seem to be infinite from within, but from an outside perspective we're bounded in spacetime, just like the Earth-centered view of the universe is bounded by our cosmic event horizon.

THE HEAT DEATH OF THE UNIVERSE

Another plain fact about our experience is that it's pinned to a moving front in time that leaves present moments in the past as it flies forth into the future like an arrow. The trajectory of this arrow of time has a puzzling manifestation in physics. Time flies toward what the early theorist of thermodynamics Ludwig Boltzmann called the "heat death" of the universe. Entropy increases, things fall apart, and we age and die. This fact of life deserves a little more explanation.

According to Boltzmann, the entropy of a given state of a physical system at some time is (logarithmically) proportional to the probability of that state relative to the other states the system could have been in at that time.

The tricky part of Boltzmann's definition is the set of other states a system could have been in. Specifying the set involves what philosophers call counterfactual reasoning. We need to consider what's possible in the sense of being allowed by the laws – in this case of physics.

The problem is that in a classical universe the dynamical laws are deterministic, which means that given a specified state of the universe at time 1, its state at any later time 2 is fixed, determined, and predictable (in principle) from the laws of physics. A Newtonian universe is a piece of divine clockwork that once wound up at Creation simply ticks on relentlessly, eternally bound to follow the divine laws. Einstein's revision of the story didn't undo this determinism; it only added that it was impossible to synchronize all the clocks.

If the entropy of a deterministic system is to be anything other than zero, we need to define nonzero sets of possible alternative states of the system. Boltzmann saw that the laws used to define the state of a system in thermodynamics were phenomenological – they concerned the appearances of the system but not the underlying mechanisms at play. The laws were in terms of temperatures, pressures, and so on, whereas according to what was then called the atomic hypothesis the underlying reality was that vast numbers of tiny atoms and molecules interacted to give rise to those appearances. The phenomenal properties defined what Boltzmann called the macrostate of a system, whereas the atomic hypothesis defined a vast number of microstates that were consistent with that macrostate. He defined the entropy of a given macrostate by appeal to the number of possible microstates that could give the appearance of being in that macrostate.

Time's arrow is therefore an appearance generated by our fixation on the phenomenal properties of systems, such as our own bodies, that age and die in time. If we could zoom in on the microphysics, or the nanophysics or whatever, we'd find no evidence of the unidirectionality of time. Everything would be reversible, as classical determinism predicts.

But this is lamentably not feasible. Consider the weather. Meteorologists can't predict it accurately into the future even with powerful supercomputers, because the weather systems on Earth are chaotic. As the meteorologist Edward N. Lorenz discovered, tiny atmospheric fluctuations below the threshold of measurement can very quickly snowball (if that's the right word on a warming planet) to produce huge weather events. Crudely, chaos is an unpredictable mess where science as we know it loses its grip, and chaotic systems quickly devolve into chaos. They resist exact predictions – and if you look closely they're everywhere in nature.

When it comes to our ability to make good predictions, entropy is a fact of life, however deterministic the underlying physics might be. We live our lives in a world of appearances.

Such a world can appear chaotic and display rising entropy even if there's a stable order beneath the chaos. This sad fact overshadows the yearning for an eternal order that Einstein and Gödel shared with Plato.

Recognition of entropy and chaos marks a big change in the story of physics, for it accepts that epistemology and ontology often part company. The contrast between the two has been familiar for many centuries among philosophers. Ontology is the study or the theory of what is or what exists, whereas epistemology is the theory of knowledge. Generally, what exists as fact about some topic and what we know about that topic are two very different things.

Today, we're used to the idea that many of the things we measure are conventional approximations or simplifications that disguise or gloss over a mind-numbingly complicated and counterintuitive underlying reality. The epistemology of the comfort zone of our everyday lives is worlds away from the ontology of the fundamental reality revealed in physics and math. Yet there's an important link between the two. We aim to know the truth, on pain of sinking into fiction and fantasy; and the truth must remain accessible in principle, on pain of sinking into metaphysics and fantasy.

Here is where a brief homily on truth might be pertinent. Epistemology concerns what we know or can prove; ontology concerns the truth of what we know or can prove. To declare that something is true is normally to say that it matches the agreed facts or is provable from agreed truths. It's not to refer darkly to mystic claims of eternal truth.

Among analytic philosophers, the widely agreed paradigm for making sense of truth claims is due to Alfred Tarski, who built on Gödel's incompleteness theorems to prove that the set of truths for a formalized theory including arithmetic was undefinable. Tarski's theorem used a simple parsing of truth claims: "Snow is white" is true if and only if snow is white. That is to say, as Quine helpfully explained, predication of truth is a device of disquotation. We assert that a statement is true by saying what it says, either in so many words or in an equivalent way, such as in paraphrase or translation.

Tarski's aim with his theorem was to advance model theory in formal logic, where a model of a formal language is the ontological structure by means of which we define the truth of statements in the language. We can double down on Tarski's formalistic approach by defining meaning for such a language: "Snow is white" means that snow is white. These definitions make explicit what might otherwise be forgotten, namely that both truth and meaning are linguistic concepts.

The relevance of this homily to epistemology and ontology is that whereas epistemology concerns what we're entitled to say, ontology concerns the truth of what we say. Looking at any debate in theoretical science that continues for years or decades, we see what looks like a tag-team ping-pong match between epistemology and ontology, as successive debaters argue back and forth with claims and counter-claims about the issue under debate, all the while going ever deeper into the truth behind it. In the process, scientists develop better and better models of the issue.

The early or simple models scientists build, either to get started or to gloss over complexities their computers can't handle, are what we call effective models. An effective model is not yet the final truth. It's a stage set or a Potemkin model, masking a deeper reality that might be quite different and run according to different laws. But once the idea of an effective model is there, we can reasonably ask whether physics is a sequence of effective models all the way down. We can ask whether below the atomic model, below the standard model of particle physics, below even string theory or a quantum information model, there are more, if not forever then at least until we hit rock bottom with a primal bipolar model poised on the ouroboros.

ALL MEN ARE MORTAL

The ideas we've reviewed from logic and set theory are just what we need to describe the psychological take on time in a form that may be useful in psychophysics. The description also raises issues that lead us from classical to quantum physics.

The psyche, the mind, is a zone of consciousness where we experience a "manifold of sensation" (Kant's phrase) located now, in the present, at the temporal boundary between the past and the future. Time is fundamental to consciousness.

The boundary between past and future moves in a way that transforms the 0 of future being into the 1 of present existence, and the 1 of the lived present into the 0 of past existence. Alternatively, it transforms the 0 of fresh being into the 1 of existence on top of a pile of past layers of existence that fade to 0. The ambivalence is inevitable, intrinsic to time as we know it. We experience time as pure change.

Time eludes logic as we've classically defined it. Logic starts out from the fixed points of 0 and 1 and then proceeds to define allowed moves that preserve them as separate. Logic eternalizes a moment in time and then articulates a consistent worldview – or a consistent state of things – relative to that moment, or from the perspective of that standpoint in time. When a moment is one of cosmic clarity, like those Einstein might have enjoyed from time to time, the Olympian rapture of glimpsing eternity is surely seductive.

Scientists prize consistency for good reason. They aim to develop models of reality that survive recalcitrant experience and produce reliable predictions of their future experiences. The extraordinary fact that nature reveals to us innumerable exquisitely precise patterns at every turn enables scientists – against all naïve expectation – to define mathematical models that actually work. This amazing fact was a source of wonder to Einstein. But he wasn't alone: Ever since Galileo, scientists have said in various ways that God is a mathematician. Mortal mathematicians who enjoy unexpected success in exposing the secrets of nature may be forgiven for imagining they've been granted divine revelation.

Returning to temporal change, the onward march of time propels us from one view of the world to a different world-view, or from one consistent state of things to another. By applying our 0–V metaphysics, we can model this process in logic. But first, we need to review more logic.

Most people today have some acquaintance with Boolean logic, which is the logic of 0 and 1 used to program all classical computers. This is the algebraic version of propositional logic, or the logic of atomic sentences that may be true or false. The logic combines these sentences with such operators as NOT, AND, and XOR, to form compound sentences, which have truth values (0 or 1) that we or a computer can calculate using algorithms based on simple rules.

But classical logic goes further. Years before he worked on set theory, Frege invented what we now call predicate logic by applying the mathematical notion of functions to the atomic sentences of propositional logic. Recall that a function $F(x)$ of x is a map from an object x in its domain to another object y, which may be x itself or something quite different, such as a truth value, but must be uniquely determined for each x.

We can explain this by translating a sample sentence that Aristotle used to discuss syllogistic logic: "All men are mortal." Within this sentence, we discern a predicate, "( ) are mortal" or "( ) is mortal," and a subject, denoted by the noun phrase "all men," that refers to certain things, namely men. The sentence says that those things "fall under" (Frege's phrase) the concept denoted by the predicate.

We can unpack the sentence further by invoking a second predicate, namely "( ) is a man," and using a variable x to stand for anything, any existing thing, so long as x stands for the same thing throughout the sentence. We can now transcribe our sentence thus: "For all x, if x is a man then x is mortal."

The function notation comes into play when we code the predicates "( ) is a man" as $F(x)$ and "( ) is mortal" as $G(x)$, where $F(x)$ and $G(x)$ are functions that range over things x. Substituting a name for x results in sentences that may be true or false. In this case, $F(x)$ maps "x is a man" to a truth value (0 or 1) when a name is substituted for x, and likewise $G(x)$ maps "x is mortal" to a truth value. Now Aristotle's sentence becomes the formula "For all x, if $F(x)$ then $G(x)$."

You can be forgiven for seeing no gain in this transcription, but in fact it was soon recognized as the biggest breakthrough in logic since Aristotle. Frege had translated the claim "All men are mortal" into the claim "For all x, if x is in the set of men then x is in the set of mortals," which in his predicate logic implies the lesser claim "If Socrates in in the set of men, then Socrates is in the set of mortals," which we can say is true if the set of men is a subset of the set of mortals.

Until then, no one could prove the validity of Aristotle's syllogism: "All men are mortal, and Socrates is a man, therefore Socrates is mortal." But now, if we let the name constant s stand for Socrates, we find that "For all x, if $F(x)$ then $G(x)$"

is the first premise, “$F(s)$” is the second premise, and “$G(s)$” is the conclusion. Frege suddenly found it possible to prove that this inference is valid. He ushered a whole new continent of logic into view.

Frege used his new formalism to express and develop his set theory, and the logical power and precision it introduced was the enabler for the entire study of axiomatic set theory. Years later, proof theory, model theory, and more followed. But the details are endless and out of place here.

Predicate logic is also called first-order logic, because its quantifiers – expressions like “For all x” – range over objects that all have the same status. Second-order logic would let us quantify over the concepts denoted by predicates with a second style of variables, to yield doubly quantified sentences like this: “For all x and for all F, $F(x)$ or not $F(x)$.”

In general, second-order logic is too powerful for its own good, for a hard reason: The concepts denoted by predicates can be identified with classes. In the usual notation, $\{x| F(x)\}$ is the class of objects x such that $F(x)$. A quantifier ranging over classes F would treat them as existing things, and soon we’d be faced with paradoxes like those that befell Cantor and Frege. To stay safe, we do without second-order quantifiers and treat such classes as proper classes.

A compelling picture now emerges of how any first-order logic maps into the set theory of the cumulative hierarchy. The quantifiers range over all existing sets of a certain rank and below. We map any objects that don’t look like sets as Quine would, by simply mapping them to sets in some logical way. Finite domains of objects map to sets up to a certain rank, and all the possible groupings of those objects by predicates map to members of the power set with the next rank up in the hierarchy. This power set is a subuniverse of the universe V.

Let's call any such power set a V-set. For finite domains, our compelling picture is that any first-order logic has a formal model in a V-set.

For infinite domains, which are usually the ones of interest, we need to be more subtle. We say the union of all the power sets we get from sets of all finite ranks is the V-set whose index is the limit of the finite ordinals. And so on.

All this may seem horribly technical, but its relevance to the psychology of time passing can now be read off directly. The cumulative hierarchy of sets is defined along a dimension that invites comparison to a time dimension. Its ranks appear one by one at progressively later points along that dimension. So, if a system of logic has a model in a certain rank and allows quantification over sets of lower rank, the logic presupposes a move forward along the timelike dimension. To do logic on sets of a certain rank, we have to step up a rank to find the power set that lets us model that logic. Following the time metaphor, we take a step forward in time even when the logical inferences we draw are temporally flat. We do logic on one level, but we step up a level to see what we're doing.

We can describe this step upward as what Quine called a linguistic ascent. We use a simple trick to let the model theory for a rank treat the sets of that rank as linguistic entities and have them stand for their own syntax, then interpret stepping up a rank as saying things about the language items. If the model theory is expressed in the metalanguage for them in this way, stepping up a rank is equivalent to developing a Tarskian truth theory for the language in the previous rank. Some fifty years ago, the logician Saul A. Kripke developed a recursive Tarskian truth theory in ranks forming a cumulative hierarchy in just this way, using repeated linguistic ascents to go from metalanguage to meta-metalanguage and so on, and then said they all formed a single growing language and tried to find a stable limit to the growth. We can also see what he built as a tower of languages reaching up into infinity – or indeed as a model for an ontico-epistemic dialectic. Such a dialectical logic goes beyond the logic of Aristotle and even beyond the logic of Frege. All men are mortal.

FROM LOGIC TO PING-PONG

Gödel had provided a precedent for the linguistic ascent that Quine and Kripke used to such dramatic effect. Gödel's contribution is worth describing more fully because it forms the basis for a theorem proved by Alan Turing that we cite later when we discuss the difference between human beings and intelligent machines. That comes later – here we just offer a brief overview of Gödel's logic to underscore the importance of his famous incompleteness theorems.

Gödel began his career by proving the completeness of predicate logic. In our terms, he developed a simple model theory for it. He modeled the formal syntax of the logic in a metalanguage built using the same syntax, and proved that the strings of syntax in the object language representing proofs in the logic mapped one-to-one to truths in the metalanguage: What was true was provable, and vice versa.

That was the easy part. Next, he considered the theory of arithmetic, where the infinity of numbers made the challenge more difficult. Gödel instrumentalized a variant of Cantor's diagonal argument to define a formal statement in arithmetic that could neither be proved nor disproved in arithmetic. He coded both the language (using Fregean syntax) of arithmetic and the metalanguage for arithmetic into arithmetic using a code of his own invention. He then defined, first, in the object language, what was in effect an alphabetic list of all the proofs in arithmetic, and second, in the metalanguage, a sentence he could interpret using his code as reading: "For all numbers *x*, *x* is not the number of a proof of this sentence."

Think about it: This sentence is true if it's unprovable and false if it's provable. If arithmetic is consistent, this sentence can't be among its theorems. And if the syntax of arithmetic contains a formally correct sentence that remains undecidable, then arithmetic is incomplete. The big result, for which Gödel has earned celebrity status, is this: If arithmetic is consistent, then it's incomplete.

Gödel's second incompleteness theorem reapplied the first theorem to prove the undecidability in arithmetic of a claim we can call X and interpret as follows: "For all *x*, *x* is not the number of a proof of a contradiction." He then argued that if (a) arithmetic is consistent and we can prove it in arithmetic, then (b) we can immediately prove that claim X is a theorem; but we can't prove (b), so the premise (a) is false. To prove that arithmetic is consistent, we need to go beyond arithmetic by working in a stronger metatheory.

All this is still abstract math, and the timelike dimension in set theory is not yet the time of the physicists. We still have some work to do to flesh out the story behind the central and most striking fact of psychology, namely that our experience takes place in time. Let's start by considering the temporal process by which we become aware of new information.

People typically make assertions to say something they take to be informative. They presume that the persons to whom they direct their assertions don't yet know the information in question. Those persons are moved by the assertions from an initial state of information about the topic in question that's somehow gappy or incomplete to a final state that's less so.

The new information transports them between two epistemic states, one before and one after, where an epistemic state is a state of knowledge.

If these two states claim to be the facts about the topic in question, there's a rhetorical sense in which the two states contradict each other. A prior state A defined by fewer facts is replaced by a later state B including further facts. Something has changed – in time. If we say that state A represents our knowledge of the facts and state B represents the full facts, A is an epistemic state and B is by contrast an ontic state.

Clarity here is not served by taking the contrast as absolute. Our knowledge in some field always has a current best state, which we have no practical choice but to accept as the factual state of affairs. But as science advances, our knowledge moves on, and soon what we thought was an ontic state is devalued in our estimation to a merely epistemic state. Generally, what Hegel and his contemporaries would naturally have called a dialectical interaction between these two poles then emerges. Today, we'd rather describe such an interaction between two contrasting poles as an example of runaway evolution. Either way, the logic is clear enough. What looks like an ontic state at time A can be revealed as an epistemic state at time B.

In this way, the temporal transition from state A to state B can be seen as part of an ongoing interaction between ontology and epistemology, where each state can be seen as either ontic or epistemic, depending on its context. Formal logic applies to each state A or B considered in isolation. The jump between states can be seen abstractly as a jump along a dimension of becoming or can be seen more concretely, depending on the context, as a step forward in time. Formal logic is blind to the jump. It sees a flat state of affairs and tells us what we may deduce from that set of flat facts. It ignores the ping-pong paradoxes of epistemo-ontic equivocation. In terminology now deprecated by philosophical fashion, it ignores dialectical change by denying the evolution of truth claims. For short, let's call such a ping-pong or dialectical run of epistemo-ontic flip-flops a runoff.

We can now relate all this to the cumulative hierarchy of sets. With a suitably Quinean squint of the eye, the states A and B of information can be mapped to specific ranks of sets. Then the move from state A to state B is a move up in rank. We can gloss the process and say the move from rank to rank up the hierarchy is an abstract representation of the dialectical runoff between ontology and epistemology lying behind any informative exchange of assertions.

REFLECTING THE ULTIMATE TRUTH

The ping-pong story invites a deeper representation. The tower of ranked V-sets forming the universe V is a stack of ever more convincing partial or provisional versions of V. Each V-set looks like V from below, but then we get over it, and suddenly it doesn't anymore. Depending on how we map sets to something more amenable to our imagination, what we get is a series of ever clearer or sharper impressions of V. It's as if the tower is a series of low-res versions of a JPG image that get more focused as we crank up the resolution. As the pioneer Cantor reminded us, the ultimate universe V of sets is transfinite and overwhelms any attempt we make to define it once and for all, so the V-sets are all too fuzzy to do justice to the ultimate truth.

Set theorists have invented a class of axioms to represent what's going on here. They call them reflection principles, and they're just what we need to tell the ontico-epistemic story more clearly. A reflection principle posits the existence of a V-set that reflects V for a given set of axioms. In other words, all the axioms are satisfied within that V-set, so for that theory V is reflected in that V-set. For the axioms of ZF set theory, a reflection principle posits the existence of what's called the first inaccessible cardinal. This claim probably needs a few words of explanation.

We saw that in transfinite set theory, there are two kinds of numbers: ordinals and cardinals. For finite numbers, ordinals and cardinals coincide, but for transfinite numbers they don't. Ordinals are always countable, but cardinals soon become uncountable. As Cantor proved, the cardinality of the power set of all finite sets, which he called aleph-0, is still countable, but the power set of any set with cardinality aleph-0 has a higher cardinality. The cardinality of real numbers, hence of the continuum, is uncountable, strictly greater than aleph-0. Generally, for any set x, the cardinality of the power set of x is greater than the cardinality of x.

A spot of history may help this make more sense. Gödel spent his early years at Princeton trying to prove or disprove Cantor's continuum hypothesis. Cantor had guessed that the cardinality of the continuum was aleph-1, the next transfinite cardinal after aleph-0, but he was unable to prove or disprove it. The hypothesis remained in limbo until Gödel had a crack at it using the tools in ZF set theory (or rather, to be exact, NBG theory with proper classes).

Gödel proved that Cantor's hypothesis is consistent with ZF set theory. He did so by defining a constructible universe L, which was like V except that it contained only sets he could construct with the tools available in ZF, and showing that the hypothesis is true in L. Set theorists then argued for decades over whether or not V = L was true in ZF, until Paul Cohen proved that the negations of V = L and of the hypothesis are also consistent with ZF.

That was history, recited only to dramatize questions of cardinality. Because the mathematical excitement of set theory resides mostly in questions of cardinality, theorists tend to discuss axiomatizations of set theory mostly in terms of the highest cardinality of the sets they comprehend. It's as if your bragging rights as a set theorist are proportional to the size of the biggest tower of cardinals you can erect. Since any cardinal comprehended already in ZF is accessible, a principle asserting the existence of an inaccessible cardinal is enough to take us strictly beyond ZF.

You might reasonably protest: If ZF is enough set theory for the foundations of mathematics, what's the point of trying to go beyond it? The natural answer invokes Gödel's second incompleteness theorem. In set theory, a consistency proof for a theory reflected in some V-set requires going up a rank and positing the existence of that V-set. In other words, to prove the consistency of a set theory, you need to show that all its sets are members of some V-set.

This is remarkable. It implies that to prove a theory of V is consistent, you have to prove it's satisfied in a V-set that's not V, by definition. So, to emphasize the point, there is no theory of V. There are only theories of bigger and bigger V-sets, with no end in sight. Reflection principles and consistency proofs do no more than reinforce the message.

In short, we need a principle asserting the existence of an inaccessible cardinal to prove the consistency of ZF; so if ZF is the foundation of math as we know it, that principle is the least we need to sleep easy at night.

Though Gödel had proved already that there is no fixed and final theory of math (essentially by coding difficult sentences in any theory that could only be decided in a stronger theory), the Sisyphean treadmill of endlessly building towers of V-sets reflecting a paradoxical universe V where closure leads to the jaws of the ouroboros is more vivid.

Philosophically, we can understand the situation in a more pragmatic way: The truths of mathematics are not so much eternal as so far abstracted from practical or temporal life that their origins are hidden. Pushing for yet more abstraction with ever more ambitious reflection principles merely accelerates our trajectory headlong into a mushroom cloud of smoke and mirrors that veils our psychedelic trip into Cantor's paradise. Bigger V-set depictions of V are more perilous, and since we have to stop somewhere, we might as well stop at a relatively humble level, for example one that suits the Quinean mapping we have in mind to a more concrete ontology.

The unfinished tower of V-sets is a logical resource we can help ourselves to in novel ways. Philosophers used to say set theory went beyond logic because it carried big ontological commitments, namely to something like an inaccessible V-set and the sets that came with it. They imagined that set theory required us to comprehend as many sets as the most ambitious reflection principles could seem to give us, held back only by our confidence in the formal syntax needed to define them. And, quite rightly, they hesitated to make logic a hostage to mathematical adventurers.

But the philosophers' assumptions about set theory were unwarranted. Since set theory must stop short of V, there's no logical reason not to stop a long way short, for example at the first rank where V = {0}, to leave us with the bare universe of 0 and 1 for Boolean logic, or more bravely to stop at finite ranks and deprecate wild talk of infinite sets, or more boldly to stop at countably infinite sets and refuse to comprehend the classical continuum. Each of these “little” logics reflects the truth better than before, but never perfectly.

A historical aside can underscore for us the implications of accepting that such towers of V-sets are integral parts of the machinery of logic. Kant proposed both that we see arithmetic as logic and that the line of natural numbers reflects in its infinity the “intuition” of time. Recall that time for him was a form shaping the manifold of sensation. We can’t follow Kant now that we’ve moved on from Euclidean geometry, but we can still say time comes within the purview of logic.

Classical logic, as formalized for example in predicate logic, offers what appears to be a timeless view of reality. But the better line to take is that it offers an eternalized view of a state of affairs that nevertheless carries an implicit timestamp. Even Einstein’s apparently eternal and deterministic block universe, with its four-dimensional curved spacetime, is stamped with the dated epistemology of its mortal author.

Despite all his best efforts to do so, Einstein was unable in his Princeton years to get his head around the implications of quantum theory, which apparently compels us to develop a conception of time that’s incompatible with a deterministic block universe. But what is quantum theory?

SUMMARY

Let’s close the chapter by summarizing the more general view of time suggested by our take on set theory.

A custom-sized tower of V-sets between 0 and V offers a flexible view of time. We can split moments of time more or less finely by interpolating bigger or smaller towers of V-sets between the endstops of the 0–V loop. We can also impose minimal time increments by setting their endstops at 0 and 1.

This unlimited zoomability reflects the idea that time is pure change. That idea is independent of the details of any more specific physical theory of time we choose to develop and seems to be an eminently logical one.

Logic is more than the science of valid inference. It's also the science of what to do when paradox looms and a frame is needed to accommodate a response. A theory of becoming that lets beings emerge from nothing, with a logical runoff to escape the jaws of the ouroboros, with those beings ready to be regimented into ranks of existing things, is the least we can ask of a logic worth calling a science.

The tower of V-sets improves on the number line as an abstract model for time in a nondeterministic physical theory by enabling us to model at each moment the space of possible futures revealed by such a theory. Since a V-set is a power set containing all possible sets built from sets of lower rank, it offers the ontic width to entertain the being of a twilight zone of possible worlds.

Einstein never accepted that we live in a quantum universe where the parallel existence of such worlds comes into play. Quantum theory involves a deep dive into the whole business of ontico-epistemic runoffs.

We explore quantum theory in the next chapter – and find that the logic of becoming is helpful.

PLANCK'S HAIKU

Quanta are tiny,
but if spacetime is quantized,
below lies chaos.

STATES

The quantum revolution began in Berlin in 1900. Max Planck rescued researchers into thermal radiation from a problem by suggesting an equation that included a new constant.

We call this new constant Planck's constant, h. It sets the characteristic granularity of quantum phenomena and is basic to quantum theory. Technically, h is the quantum of action, where an action is a quantity of work done over a period of time, and it's a miniscule fraction of a joule second, where joules measure energy or work and seconds measure time.

Planck's constant got a boost in 1905, when Einstein used light quanta to solve a puzzle about photoelectric emission. What we call a photon is a quantum of energy E carried in an electromagnetic wave with frequency f. Planck defined light quanta as having energy $E = hf$.

Next, in 1913, Niels Bohr proposed a quantized model for the hydrogen atom. Experiments had shown that atoms are mostly empty space, suggesting the idea that electrons orbit the nucleus like planets around a star. Bohr calculated that if the electrons' angular momentum were quantized, their orbits would be stable – and match the experimental data.

Bohr's idea was no more founded in a physical theory than those of Planck or Einstein, because none of them could say why things were quantized. But people were thinking.

In 1924, Louis de Broglie proposed the principle we know as wave–particle duality. Puzzling over observations revealing that beams of photons or electrons interfered and diffracted like waves, he suggested that electrons or photons, or indeed anything of comparable size, could appear as either waves or particles, depending on the experimental context in which we viewed them. Einstein liked the idea.

In 1925, Werner Heisenberg digested all these ideas in his matrix formulation of quantum mechanics. He regarded his theory as phenomenological in the sense that it aimed only to describe and predict the phenomena, with no further pretense at explaining why or how they were so.

Guided by Max Born, Heisenberg found that his quantum theory implied a fundamental principle: The smallest residual uncertainty there could be in the measured values of the actions defined by pairs of "complementary" properties of a system, such as energy and time, or position and momentum, was approximately h. Heisenberg's uncertainty principle was a fundamental novelty in physics because the ontology of the theory forced it; the "uncertainty" wasn't merely an epistemic limit that a better measurement could overcome.

In 1926, Erwin Schrödinger proposed a strange new wave equation. His wave theory was easier to work with than matrix mechanics, but it introduced a mysterious wavefunction with values in the complex plane (with points $x + \mathrm{i}y$, where i is the square root of minus one). Three spatial dimensions now gave us six formal dimensions – for each particle!

The wavefunction spreads out in ripples from a quantum event and helps us to predict the outcomes of measurements of further events following that event. If the first event is the emission of a particle in a system, the measurement might be whether the particle is detected at a given point downstream from the emission. The wavefunction at that point has an amplitude (equal to the length of vector $x + \mathrm{i}y$), whose square (the Pythagorean sum of the squares of x and y) gives the probability of detecting the particle there. This rule shows that the wavefunction is just a code for describing a probability wave – rather like a crime wave, as a genial Richard Feynman later said, where crime locations plotted on a city map can let the cops estimate the probability of future crimes.

The two rival formulations of quantum mechanics, namely matrix theory and wave theory, turned out to be equivalent in their experimental predictions, but both had a big weakness. Neither was compatible with special relativity and its idea of spacetime with a speed limit.

In 1928, Paul A.M. Dirac fixed the problem. He formulated a relativistic quantum mechanics he called quantum electrodynamics (QED). He defined his own algebra featuring state vectors describing the state of a quantum system as a superposition of the possible states it might pop into following a measurement. QED had the required Lorentz symmetry and turned out to be a good theory of electrons and photons, but it had its own big weakness.

In the jargon of the trade, QED wasn't "renormalizable," which meant that nasty infinities popped up where they were least expected. The problem took some years to solve, but in the end three theorists, working independently, came up with three equivalent solutions. One of them was Feynman, whose colorful character ensured that his solution gained fame. His later brief introductory book *QED* gives an admirably nontechnical account of his key idea.

QED is often touted today as the best and most successful physical theory of all time. It provides the background theory for transistors and lasers and hence all of electronics and the modern digital world, as well as the deeper foundations for chemistry and biology. As ever, Feynman put it well when he said that QED is the theory of everything except gravitation and nuclear phenomena. Even our modern theory of nuclear phenomena, namely quantum chromodynamics (QCD), has a similar form to QED.

QED and QCD are both quantum field theories. The idea of a field goes back to Michael Faraday, who illustrated it by sprinkling iron filings onto a piece of paper held over a bar magnet to let the magnetic field pull the iron filings into a swirly pattern. Faraday's lab work was crucial for Maxwell's equations, but his idea of a field was seminal. Fields are not only useful in classical physics but are also the inspiration for the quantum fields that dominate physics today.

A quantum field is something that has a value at every point in spacetime but typically pops up as one or more particles. The field has a wavefunction that enables us to calculate the probability of detecting a particle at some location. Each kind of particle (or wavicle, as some used to say) has its own field; for example, we might imagine electrons as excitations (as we now say) of the electron field, and photons as excitations of the electromagnetic field.

In quantum field theory, the particles have no permanent or independent life, but instead are created and annihilated via interactions with other particles. Each particle is surrounded by a cloud of virtual particles that defies our imagination and reduces us to depicting their interactions in an endless series of Feynman diagrams when we try to calculate the sum total of their effects on what we observe. Such workarounds are symptoms of the fact that the fields are the main things, and the waves and particles are just fictions we grasp as props for our imaginations. As usual in theoretical physics, the ground truth is in the math.

Particles come in two main kinds, named after the kinds of statistical behavior they display. Fermions (such as quarks and electrons) remain distinct from each other and can't occupy the same quantum state when they're next to each other. By contrast, bosons (such as photons and gluons) convey forces (such as the electromagnetic force and the strong nuclear force) and lose their distinct identities when they reinforce each other in coherent collectives that behave as waves (as in a laser beam). The contrast between the Fermi–Dirac statistics of fermions and the Bose–Einstein statistics of bosons boils down to a small difference in their wavefunctions, reflecting the fact that bosons have integral spins (0, 1, 2, and so on) whereas fermions have odd half-integral spin ($^1/_2$, $^3/_2$, $^5/_2$, and so on). We'll find that the bosonic behavior of photons plays an important role in the quantum view of consciousness we introduce in the fifth chapter.

To round off the history, once we'd gone beyond QED to unify electromagnetism with the weak nuclear force, defined QCD to explain how quarks and gluons behave in the nucleus, and developed a theory of the Higgs field to explain the rest mass of particles, quantum field theories made all the right predictions and showed the right symmetries to form what we now call the standard model of particle physics. It deserves our admiration – it's a historic achievement.

Notice the holdout – gravitation. Were it not for Einstein's classical theory of curved spacetime, we'd have a quantum theory of everything by now.

POPPING THE BUBBLE WRAP

Despite the glory of the standard model, there are several deep and difficult questions it leaves completely unresolved. Many of these were already questions in the early years of quantum theory, and all the work done since then has only deepened the sense of paradox they revealed.

Right from the start, Einstein and Bohr were debating the new ideas. Einstein was disturbed by the loss of predictability in physics. We were reduced to saying the wavefunction for a system "collapsed" when a measurement was made and that the best we could do was predict the probability of this or that outcome. Einstein said "God does not play dice" and tried to find problems with the new theory, but Bohr defended it and usually found a way to win the exchanges.

Bohr developed his own distinctive interpretation of the theory, which was known as the Copenhagen interpretation. Its main contribution was to emphasize the epistemological limits on what we can say about the presumed ontology of physics. Bohr would say it was wrong to think that the task of physics was to find out how nature is; its task was rather to map out what we could say about nature. In other words, we have another ontico-epistemic runoff at the heart of physics: Quantum theory repeatedly bestrides epistemological hurdles that classical theory tries to ignore.

Classical physics projected an idealized model of reality as the ontology that made its epistemic claims true or false. This worked for a while, but got difficult when in thermodynamics Boltzmann's definition of entropy distinguished macrostates from microstates. Quantum theory takes the problem of the gap between ontology and epistemology to a new level.

In 1935, Schrödinger described an odd consequence of his wave mechanics. The wavefunction correctly predicts the outcome of simple experiments and conforms to wave–particle duality as well as one could wish. But it implies that properties of a system which have not been measured leave the system in a superposition of states. For example, if you put a lump of radioactive material beside a detector into a box and leave it unobserved for a time equal to the half-life of the lump, the contents of the box evolve into a superposition of two states, (1) decayed lump with triggered detector and (2) undecayed lump and untriggered detector, each with an equal probability of popping into existence when you open the box.

Later work showed that such superposed states really do remain in being until you make a measurement. The problem, as Schrödinger wryly observed, is that if the lump's decay also triggers a hammer to fall on a vial of poison gas, breaking it, and you also put a cat in the box, which dies if it breathes the gas, the system remains in a superposition of live cat and dead cat until you open the box! He thought this was an absurd consequence of his theory.

It got crazier. In 1961, Eugene P. Wigner followed though by describing a scenario now dubbed "Wigner's friend." In terms of the cat, imagine the friend opening the cat box in a lab that's sealed off from Wigner, who's working in his own lab outside it. The cat begins in a superposition of dead and alive, but then the friend opens the box and finds it dead or alive. But for Wigner, the cat is still in its quantum state, and remains so until he opens the door to his friend's lab. What's changed is that his friend is now in the superposition too. And if Wigner and his friend had changed places, so that his friend didn't know what Wigner found when he opened the box, Wigner himself would be in the superposition! For a classical theorist, this was just ridiculous.

We now know that these scenarios are impractical. In 1970, H. Dieter Zeh showed that an effect later called decoherence prevents any superpositions in a warm and poorly isolated box from growing anything like big enough for cats or people, because random interactions with their environment act like measurements to pop them well before then. This solves the cat problem in practice – but not in principle.

Decoherence aside, superpositions of states in radically unknown abundance may well lurk beneath any macrostate we ourselves occupy. How can we build a theory of mind on that basis? Classical reality is lost without trace if vast numbers of microstates lie waiting for me to decide my destiny before one state pops into existence.

An answer of sorts begins to emerge when we ponder the quantum phenomenon of entanglement. But it only confirms that we have to ditch the classical view of reality.

Only months after Schrödinger's cat paper, Einstein coauthored a paper with Boris Podolsky and Nathan Rosen that introduced a paradox they thought refuted quantum weirdness. The trio imagined an experiment where an event on a lab bench emitted two entangled particles in opposite directions. Their entanglement meant that the values of certain of their properties were correlated. For example, if we measured spin up for particle A, we could be sure that particle B would show spin down if we measured it. So far so good.

A problem emerges, said the trio, when we simultaneously measure a different property of particle B. We can choose to measure two complementary properties – say the position of particle A and the momentum of particle B. The uncertainty principle says we can't simultaneously get exact values of both properties, because each measurement pops the respective entangled properties of both particles at once: If A has position x then B has position $-x$, and if B has momentum p then A has momentum $-p$, so both of the implied products, $x(-p)$ for A and $p(-x)$ for B, have exact values, which violates the minimum fuzziness of about h imposed by Heisenberg's principle. The problem here is that if entanglement wins, uncertainty loses; and if uncertainty wins, entanglement loses. Something has to give.

The trio said this showed there was a hidden mechanism somehow changing the properties. Okay, let's assume that a hidden mechanism passes information about the respective states of particles A and B between them. We have no clue what the mechanism is, but given relativistic spacetime, the message can pass at anything up to light speed.

Now for the punchline: If we can make the two measurements of their complementary properties in less time than it takes for a message to pass between A and B, and if again the results are in line with quantum theory, then the presumed hidden mechanism violates the cosmic speed limit. In other words, the mechanism linking A and B violates what Einstein called locality: Any messaging mechanism that works locally does so by sending its messages along the line between A and B, and so the messages are forced by the geometry of spacetime to travel at speeds less than or equal to *c*. If tests show that quantum theory is nonlocal in this sense, then it can't be relativistic, and all bets are off.

That's the EPR paradox. The trio concluded that quantum theory is incomplete, and there must be a hidden mechanism overriding one or other of the quantum ideas here.

In recent decades, with the advent of fast electronics, we've been able to perform such experiments on entangled pairs of particles and measure them simultaneously when they're far apart, before a message can travel at light speed between them (when their relativistic separation is spacelike, in the jargon). The variants on such tests have been endless, reflecting ever more elaborate experimental protocols to plug any remaining loopholes that could tempt diehard classical realists.

We now call such experiments not EPR tests but Bell tests, after the CERN physicist John S. Bell, who in 1964 proved a theorem showing that the measured correlations in properly conducted tests would accumulate statistics demonstrating with mathematical certainty whether the quantum view was correct or instead a classical story could be concocted to explain away the results. Over the decades, the results we've accumulated in countless Bell tests are clearly consistent with quantum theory and inconsistent with a classical story.

To underscore this conclusion, in 2022 the experimental physicists Alain Aspect, John F. Clauser, and Anton Zeilinger won the Nobel Prize in Physics for multiple experiments that used Bell tests on correlated photons to demonstrate beyond doubt the existence of entanglement, as described by quantum theory, and to exclude any philosophical objections raised to date that would leave room for the idea of classical local reality to make a comeback. We live in a quantum reality.

Entangled particles show all the hallmarks of remaining in superposed states right up to the moment when test measurements pop the correlated properties into existence. Everything we know confirms the quantum story.

But what is the quantum story here? And how does it relate to consciousness? Well, the story is that systems remain in superpositions of states until the moment they're measured. And the pioneers proposed that what made measurement special, in contrast to any other physical process, was that human consciousness is involved. We pop the bubbles.

Today we can fall back on decoherence. But entanglement turns out to be a crucial phenomenon for understanding how we're embedded as we are in the world.

Entanglement is a pervasive phenomenon in nature. The early quantum pioneers were puzzled over why we live in what seems to be a classical reality when everything around us has its foundations in a quantum reality. Heisenberg decided to let reality remain bipolar by making a "cut" between us as human observers, with our labs and our measuring instruments, and the quantum systems we study with those instruments. A little later, von Neumann made his cut more sharply between feline and human consciousness.

Both of these cuts were only pragmatic responses to the philosophical fudges and obscurities within the Copenhagen interpretation, and everyone could see they were untenable from a philosophical point of view. On the other hand, they were, at the time Bohr cooked up his philosophy of complementarity, the only available responses to a simple fact of observation: We don't see superpositions or entanglements, even though they're all around us in vast profusion.

Roughly, we live in a macrostate of the world, where only close study reveals the buzzing hive of activity in the microstates that lie behind or beneath our macrostate. Our senses have been shaped by evolution to help us survive in world of familiar things that reveal their quantum granularity only to much finer investigation. The blurred macrostate we live in is in constant interaction with the microstates we study in our labs, and when we pop quantum bubbles it can change our state too, but not by enough to notice. We live in a big bubble in a world of bubbles. We don't appreciate that fact because our senses are so gross. In short, we entangle with the systems we study, which puts us into a definite state relative to them. After we've entangled with them, they look classical to us, but the quantum chaos is still there, tamed but not gone.

This brings us back to decoherence. A quantum system can exist in a coherent state, which Schrödinger called a state of minimal uncertainty (and Heisenberg described technically as an eigenstate of the annihilation operator), but in a warm and noisy environment such a state is unlikely to survive for long. The quantum process of decoherence is analogous to the thermodynamic process of thermal relaxation that increases the entropy of a system. We can liken a coherent state to a supercold state that will soon warm up by interacting with its environment. The coherent state will evolve through a series of states that continue to entangle with that environment until the system is in a unique state relative to it. If the environment is us, it will have popped relative to us, leaving us in our calm classical world, where we're oblivious to the quantum fizzing and popping going on below the surface.

Imagine a carefully isolated qubit in a quantum computer as a bubble, and imagine an array of such qubits, prepared to run a quantum algorithm, as a sheet of bubble wrap. Now take this image and apply it to raw nature outside the computer, where such bubbles lie scattered all around like microscopic clones of Schrödinger's cat in tiny boxes, waiting for gross, clumsy creatures like us to come along and stomp all over the bubble wrap, crushing the bubbles until we've popped them all and blithely imagine we're in a flat classical landscape. That's how we've evolved to live in our quantum world.

PHENOMENAL PROGRESS

The big outcome of decades of theory and experiment, then, is that superpositions and entanglements are real. We know they are. Quantum computers depend on them, and we can already conjure the expected results out of such machines. By delicately superposing parallel virtual computations in arrays of qubits, these machines can in principle achieve exponential speedups when running complex algorithms (which need to be reversible to avoid raising entropy and collapsing the quantum states via heat transfer) compared with our classical machines (which almost always raise entropy).

The issue of entropy in computing raises deep questions. The concept of entropy has two related interpretations, and both are relevant here. The first is the thermodynamic one relating heat energy and temperature, which we understand in terms of macrostates and microstates. The second is from classical information theory, as first formulated by Claude E. Shannon in 1948, where the information content of a message is logarithmically proportional to its improbability. Shannon defined the information content of a string of n bits as the log (to base 2) of the number of possible bit strings of length n, and this log is just n. With this approach, the information entropy of a string of n random variables, each with possible value 0 or 1, is n bits. Cutting to the chase, information is negentropy: The more information you need to define the state of a system, the less entropy that state has. We need less information to define macrostates than we need to define microstates, so the macrostates have higher entropy.

We humans live in phenomenally defined macrostates of our quantum reality, where defining the microstate we occupy at any given time would be dizzyingly impractical – indeed impossible for all practical purposes to calculate exactly. This will have a useful implication later, when we seek to specify worlds of consciousness, also known as mindworlds, in terms that also relate to the virtual worlds we can construct using virtual reality (VR) technology.

Both definitions of entropy are relevant if we wish to define entropy in a quantum world, where the thermodynamic states we use to calculate the probabilities in its definition include entanglements. The challenge here is to specify exactly what evidence we call on to calculate those probabilities.

Probability is an epistemological notion. Several approaches to probability theory have been pursued, and most of them emphasize that assessments of probability are made relative to specific and definable bodies of relevant evidence. This fact amounts to adopting a Bayesian view of probability – indeed some quantum Bayesians advertise their allegiance by calling themselves QBists (pronounced "cubists").

We must accept that quantum systems entangle with their environments, but we need to take care when checking how this affects the statistics we use to calculate the probabilities appearing in the definition of entropy. John von Neumann first introduced the idea of entanglement entropy, and today it's an important concept in black-hole thermodynamics. But we can stop there – it's too exotic an issue to affect the story of mind we tell in the fifth chapter.

Let's sum up what we need to know about entanglement, entropy, and probability. We human beings are big physical systems that continually entangle with the quantum states of things in our environment. As we do so, we detect no change in our macrostate, which looks classical relative to the surface of the environment we're entangled with.

This macrostate has a high entropy relative to the detailed microstates that lie beyond our knowledge. The appearance of our moving into states of ever higher entropy as time goes by is due to our habit of living in macrostates, where multitudes of microstates lie beyond our ken, and formulating the laws that suit our human habits in terms of further macrostates. Thermodynamics assures us that any new state we're likely to move into has higher entropy, and Boltzmann said this was because the new state is compatible with yet more microstates. Given that the number of microstates behind a typical macrostate is up in the zillions (where the z-prefix is a big number), the statistics behind these probabilistic assurances are reliable, and so the law of increasing entropy is pretty firm.

The probabilities involved here are still the weakest link in the chain of deductions. They presume the equal likelihood of each member of a set of theoretically postulated microstates. From our point of view, any such projection of ontological detail is an act of epistemological bravery.

Physics is an ahistorical discipline. The anachronism that historians deplore of judging the events of past or future ages or times by the standards of the present is foreign to physics. In thermodynamics, we have a theory in classical mechanics that the arrow of time leads inexorably to the heat death of the universe. If the exact course of the logical runoff that takes us forward in time is traced in more detail, the long view looks less certain. We have a classical default view, with the heat death in prospect, and a set of caveats citing quantum surprises and so on that may well change our forecast long before the heat death. Again this is beyond the scope of this book.

We can make better sense of the business of using logical runoffs to study popping bubbles by recalling the distinction between being and existence. Before a bubble is popped, the outcome doesn't yet exist. After it, the outcome exists with probability 1, and all the unrealized alternative outcomes have probability 0. The outcome is an ontological fact, and its prior probability of popping into existence is revealed to have been an epistemological guess. The various possibilities were coequal in a superposed state of being, but now a new reality has popped into existence. The unrealized states still have their being as virtual states in an abstract space surrounding the real space of existing things, but once the bubble is popped, their status is reduced to counterfactual relative to the new reality. The virtual states in their abstract space don't contradict each other or the new reality. Our cumulative hierarchy of states of being and existence remains consistent.

The vision of spacetime that emerges from the runoff story is novel. We can't see our own future, and any claim that we can do so using our latest and best physical theories is in the mathematical limit unfounded. We can have arbitrarily high confidence in the projections and predictions that emerge from good theories that hang logically together – and we must have, for reasons aired in the previous chapter, where climate science was cited as a cautionary example – but the modeling in set theory shows how excessive confidence leads us straight into the jaws of the ouroboros.

Logically, the transition from being to existence is a jump from being a proper class to existing as a set, a jump we can call an act of ontogenesis. A new member of the ontology in that context or reference frame is born. The birth takes us up a rank in the cumulative hierarchy, but it doesn't warrant our projecting the newborn entity back into the early ranks of the hierarchy – letting it in could create loops, or short circuits in our logic, that would undermine the work of regimenting new entities into ranks that pile higher without ever reaching the psychedelic paradise of ultimate V.

Physically, the initial state of a quantum system may include a superposition, but the final state, after a measurement, sees it resolved, popped into a definite outcome. The initial and final states exist at different times. A logical act of ontogenesis is represented as a time step – with the crucial feature that something new has popped into existence. We'd be guilty of anachronism if we were to project the newborn determinacy in the system back as a hidden feature of its initial state. The story of runoffs we've unpacked here enables us to insist that all physical systems grow in time.

The step forward in time made by popping the bubble of a superposed system is a step forward in a runoff. The initial state was ontic, reflecting how things are, until the measurement, which reduced its status to epistemic, leaving the final state as the new, truly ontic state, at least until the next measurement. In this picture, every state is provisional; there's always a last, best state, but it doesn't stay that way for long. Any attempt to eternalize the final state of a physical system – "these are the facts, end of story" – risks inglorious defeat in another round of the runoff.

This dynamical picture of time is inconsistent with the timelessly conjectured time dimension of relativity theory, in which logical runoffs are nonexistent and temporal order is negotiable for events with spacelike separation. We see this in the way some quantum theorists have been forced to handle spacetime in recent attempts to quantize gravity. The contrast between classical and quantum time is deep and dangerous, but understanding it is essential to getting our heads around the theory of mind presented here.

Recasting the time of quantum theory as the physical analog of the logical dimension along which runoffs unfold marks a radical novelty in physics.

CONSTRUCTIVE INTUITION

The concept of time we've developed here to cope with the oddities of quantum theory looks intriguingly like the novelty introduced a hundred years ago in math and logic when the mathematician L. E. J. Brouwer developed an "intuitionist" approach that saw math as a free creation of the human mind. Let's take a brief look at his ideas.

Brouwer proposed that we regard mathematical reality as unfolding or growing to reflect our mathematical activity in proving theorems and exploring new fields of math. He said it was an error to think that any given mathematical statement must be true or false independently of our ability to prove or disprove it within a properly formulated theory. He said so in response to the famous mathematician David Hilbert's call in 1900 for the mathematical community to find proofs for the completeness and consistency of classical math.

We all know what happened to that call. When Russell and Whitehead wrote *Principia Mathematica*, they were responding to it by building a sound foundation from which to launch an assault on the challenge – as well as doing their duty by Frege, whose life's work Russell aimed to vindicate by reworking set theory into a typed "theory of classes" that he imagined was more rigorous. When Gödel proved the incompleteness of arithmetic, as well as of any formal system like that, as he said, of *Principia Mathematica*, he was also dealing a fatal blow to the hopes of Hilbert and others. Gödel's theorem was a shocker for them all, but not for Brouwer.

Intuitionist logic denies the law of excluded middle, which had gone largely unchallenged since Aristotle's time. The law of excluded middle says that any meaningful statement must be either true or false, with no middle option. It's a core theorem of classical logic and an indispensable premise for many widely accepted mathematical proofs. Denying the law was a revolutionary act for Brouwer.

But he tried to be cautious. Given sufficiently meaningful statements, he allowed that for any such statement A, it was not the case that "A or not A" was false, so he accepted the theorem: "It's not false that A or not A." For him, double negation wasn't the same thing as positive assertion.

Similarly, he held a dissident view of logical implication. In classical logic, "If A then B" is vacuously true if A if false, regardless of the truth or falsity of B. But Brouwer interpreted

"If A then B" as saying: "From a proof of A, I can construct a proof of B." This is a stronger statement, which can clearly be false even if A happens to be false.

Brouwer was also cautious with quantified statements in predicate logic. In classical logic, the universally quantified statement "It's not the case that for all x, $F(x)$" denying that everything is eff is equivalent to the existentially quantified statement "For some x, not $F(x)$" saying that something is not eff. In intuitionist logic, we may only assert that a property holds for some x when we can demonstrate how to find an x with that property. Brouwer was mainly moved by his logical scruples here, but he was also taking aim at set theorists, who casually talked about selecting sets from infinite sets without showing in detail how they could do so.

All these novelties spurred logicians to try to formalize the logic. Several systems emerged, but all shared what we now call a constructivist approach, using the guiding principle that we accept as true only those claims for which have a construction in view to buttress the claims.

Over the decades of work on constructive logic, as results piled up, we began to take a multilayered approach, where we could select from a menu of systems that could narrow the truth-value gap as far as we chose without ever closing it.

Any constructive logic recognizes epistemic stages in the development of math and in the kinds of proof it can handle. At any given stage in that development – in any given state of the art of math – certain things or truths can be proved, and correspondingly certain things or entities can be recognized as existing. As ever, we can order all these states along a logical dimension. We can even map the key states to the historical or temporal development of math. Again we see the evidence of an ontico-epistemic runoff.

In short, the practical reality recognized by constructivist mathematicians grows and develops to reflect their activity. Looking back with our notion of runoffs that fall short of the ultimate V, we recognize Brouwer's insight here. The time of the constructivists reminds us of the dimension that holds up the cumulative hierarchy. Like Brouwer's truth-value gap, which puts future math into a fog of unknowing and shrinks with every new proof technique in math, our logical runoff time stops in the present moment but grows rank by rank to disperse the fog veiling our future.

This conception of time may seem weird. We can develop pretty clear visions of the future, at least as it pertains to the practical challenges that attend our daily lives as well as to the rocket science of predicting spacecraft trajectories. So what gives? The answer should seem clear by now: We model the future as a virtual world, and our predictions are made within the constraints and conditionalities of those models.

Evolution has equipped the human brain with impressive skills at building intuitive models of its environment and at predicting the effects of our planned actions or of possible scenarios in the terms that matter to our physical survival and wellbeing. The activity of doing science has magnified and refined such skills into our definitive human survival asset. Scientific models are vastly better than the intuitive models that each of us grows and nurtures within our skull – yet the underlying logic is the same. We can't deny that the logic is constructive rather than classical in the sense that there's a gap between true and false which we ignore at our peril. Just like classical physics, classical logic ignores epistemology.

This gap was already implicit in our earlier narrative about the ontogenesis of sets. Given ZF set theory, does the first inaccessible cardinal exist or not? The answer must wait until we either prove or convince ourselves that ZF set theory is consistent (or fail, of course). This amounts to comprehending the V-set that reflects ZF, and that's an act of ontogenesis.

Time follows a similar logic. Whenever we think about it, the next moment of time is still hiding anonymously as one state among the crowd in a fanout of virtual states in being, awaiting the pop that will promote it into existence as a real state. This works for science at large as well as it does for you or me as we plan our calendar for next week. Such plans – in science no less than for each of us – can be as realistic and reliable as may be, but in any such case a flip from being to existence is needed to turn a virtual future into a real present state, enjoying its brief moment of glory.

Time conceived in this way is the least we need to do justice to quantum theory.

GROWING SPACETIME

Physics is too rigorous a discipline to rest content with words that don't pan out in the math. Our loose talk of runoffs and ontogenesis is all very well, but what's its cash value in real physics? We're not done until we've glimpsed an answer.

We can take our cue here from researchers into quantum gravity. Their work is technical and all but incomprehensible to outsiders, but some of their general ideas for how to tackle the big questions offer toeholds for understanding. We start with a historical prelude.

In quantum theory done Feynman's way (as outlined even for poets in his pellucid book *QED*), quantum systems evolve in time by taking the path of least action – with an odd twist. They evolve by taking all possible paths from A to B, and we calculate the outcome by taking the sum of all those possible paths and finding the action for each path (where the action is a multiple of Planck's constant *h*). If we then plot a graph to show how the actions differ, we see that the curve they form has a minimum – or more generally a stationary value – which indicates the action actually required for the system to evolve from A to B.

As always in quantum theory, we have an intuitive picture and we have the math, and the math takes precedence, but in this case the reality is somehow familiar. The evolution from A to B goes all possible ways in virtual space, so the paths are traced in superposed states of being. The winner – the state that pops into existence – is the one with the least action.

Feynman's story presupposes a background spacetime that stays fixed for the duration of the move from A to B. What happens when spacetime changes in response to the move is the key problem for quantum gravity researchers. In general relativity, spacetime curves in response to the distribution of matter and energy within it. If the different virtual states in a superposition distribute the relevant particles differently, the gravitational field around them will be different, resulting in different spacetimes.

Okay, we can say, let's calculate the sum over all possible spacetimes and choose the actions on that basis. That's hard. And the math is just one of the challenges we face when we try to develop a quantum theory of gravity. Another challenge is illustrated by the black-hole information paradox.

Black holes swallow information. Lots of qubits go in, but it's hard to see how any of them get out again. Since quantum theory conserves information, those qubits must be recycled somehow. Hawking radiation suggests a sketch of an answer, but how the outgoing radiation gets imprinted with the qubits remains a mystery. Somehow, entanglements must survive across the event horizon to carry the qubits. We can accept that entanglements are nonlocal, but this much nonlocality – between disconnected parts of spacetime – is something else. One proposal is to say that entangled pairs are two faces of a single particle, where the faces are portals to tiny spacetime wormholes connecting them across the event horizon. At first sight, this idea seems desperate.

String theory offer a different and less desperate solution. Recent work has shown that when the strings and branes the theory proposes as the real constituents of elementary particles are squeezed down the throat of a black hole, they puff up into fuzzballs that together fill the space up to the event horizon, where they can interact with Hawking radiation to convey qubits. This may seem a more promising approach, but it lies far beyond our scope here.

The desperate idea of combining entanglement and spacetime wormholes is nevertheless relevant for us here. Let's give the idea some history. It was first proposed in 2013 by Juan Maldacena, who mailed Leonard Susskind the cryptic slogan "ER = EPR" on a postcard. To unpack this slogan, recall that we've met EPR already. As for ER, it's a common shorthand for the Einstein–Rosen bridges in general relativity that we all know from science-fiction movies like *Interstellar*, where they're more usually known as spacetime wormholes. Maldacena had the idea that wormholes bypass black-hole event horizons to join the points at each end of the wormhole. Susskind helped him work out the math behind the idea, and they eventually agreed to conjecture that entangled pairs of particles could be seen generally as the two sides of a single particle peeking out from the ends of a wormhole.

We need some back story here. Max Planck had discovered long before that the trio of units comprising his own constant *h*, Einstein's constant *c* for the speed of light, and Newton's gravitational constant G, formed a set of fundamental units that could replace the standard scientific units defined from meters, kilograms, and seconds. The Planck units for space and time are tiny, but they seem to be absolute. Below that scale, spacetime is undefined. We imagine it turns into what the genial relativist John A. Wheeler called "quantum foam," where a quantum quagmire of tiny wormholes and wild virtual geometries awaits the unwary.

In terms of the runoff story, we have a spacetime that up to now looks clear and unique, but from now on fans out into a chaos of virtual states in being, bubbling in a quantum foam. By realizing a unique next state, we can break the symmetry of those states relative to each other and thus step into the next moment of time, or rather of spacetime.

If entangled pairs are the paired faces of tiny wormholes, the virtual spacetimes containing the pairs are crumpled to let the faces meet in a kiss. Quantum foam is seething with such wormholes, but when its bubbles pop, the wormholes close and leave real spacetime wrinkle-free.

The physical reality we're in pops out a smooth spacetime for us just like it pops out the particles that zip around in it. So, what lies beyond spacetime?

Researchers are working on an answer, but it's early days. Quantum information is the key to modern physics. Everything should reduce to clusters of qubits that pop into bits as we touch them, spacetime included. We're aiming to define spacetime in terms of entanglements between qubits. It turns out that entanglement entropy can give us an entirely new conception of gravity to replace the view embodied in general relativity, so this looks promising.

Naturally, such work is beyond the scope of this book.

SUMMARY

Where are we? We've reviewed quantum physics and enjoyed a mental workout on the way. We've outlined a few things we need to know before we contemplate a psychophysical theory of mind. And we've checked that the logical story still makes sense when applied to physics.

The takeaways we need to keep with us in the later chapters to grok our new theory of mind are as follows:

1. Quantum reality is fuzzy or granular, with a characteristic grain size set by Planck's constant h.

2. States of the world generally exist for us as macrostates that smooth over innumerable microstates lying behind the phenomena we observe.

3. In our worlds of consciousness, decoherence limits the growth of superpositions and ensures that what we see looks classical, as evolution intended.

4. Entanglements show that we really do live in a quantum reality, with virtual worlds facing us at each present moment, waiting for us to pop their bubbles.

5. Spacetime grows as we grow, in runoff time.

But before we set off on a round tour of the new psychology, we need to understand how the new logic changes our view of the wider science of biology.

DARWIN'S HAIKU

Life on Earth began
as humble chemical scum
a long time ago.

LIFE

Any science of mind must rest on biology. Modern biology came of age with the union of the theory of evolution with biochemistry. Its story is worth retelling.

Charles Darwin embarked on a five-year world cruise on HMS Beagle in 1831 and collected a large trove of biological specimens along the way. He then spent a few years mulling over his conclusions, until in 1859 he published the book that will forever define him: *On the Origin of Species*. It marked the birth of biology as a science.

His theory of evolution by natural selection rests on three key principles. First, the individual members of a species vary from each other. Second, the variations – or many of them – are heritable between generations. Third, more individuals are produced than can survive, leading to a struggle between the individuals in which only the fittest survive. These individuals pass on their winning traits to their offspring. Thus, in the vast expanse of geological time, new species emerge, flourish, and are replaced in turn.

The theory provides our best insight into how biological individuals and species come and go in time. It has sparked a revolution in science and society that's at least as momentous as the Copernican revolution. The Darwinian revolution has a huge impact on any viable theory of mind, as we'll see.

For all its undoubted success, Darwin's theory left some big questions unanswered. One question was how parents passed on their winning traits to their offspring. It took a hundred years to find the answer.

THE SECRET OF LIFE

At the same time as Darwin was puzzling over how offspring inherited specific parental traits instead of washing them out in some kind of average, Gregor Mendel, quite independently, found the outline of the answer.

Mendel was a monk and grew pea plants in the monastery garden. Years of patient experiments with cross-breeding led him to discover three laws of inheritance. To appreciate them, we need a few biological terms, which we can best introduce in a brief sketch of modern reproductive biology.

Multicellular organisms like pea plants (or us) are made of microscopic cells with nuclei. The cells are said to be diploid because each nucleus contains a group (whose size varies with species) of pairs of matching (homologous) chromosomes. The cells can be normal body cells (also called somatic cells) or specialized germ cells. The germ cells have the special task of forming sex cells (gametes), which are said to be haploid because each of their nuclei contains only one chromosome from each of the homologous pairs in diploid cell nuclei. The process of making gametes is called meiosis and involves the chromosomes in an elaborate dance we call recombination. In short, the germ cells produce gametes, each of which has a group of single (unpaired) chromosomes in its nucleus.

Reproduction occurs when sexual activity (via flowers or other genitalia) causes gametes from the parent organisms to fuse to form the first cell (the zygote) of a new organism. In the fusion, the single chromosomes in a haploid gamete pair up with their homologous partners from the other gamete to form the nucleus of the diploid zygote.

Each chromosome carries thousands of genes that decide the traits expressed in an organism. Variants of the genes called alleles code for variants of the traits. For each paired gene, a dominant allele wins its competition with a recessive partner for expression in the new organism.

Mendel's laws summarize all this. First, the paired alleles for a trait separate during gamete formation, so a gamete has only one allele from each pair. Second, an individual that inherits a dominant allele for a trait expresses the trait it confers. Third, the alleles for different traits sort into gametes independently of one another. These laws imply that an offspring's traits are expressed in unalloyed form and that the allocation of genes seems random, as if recombination is a lottery.

The finer details of the exact chemical mechanisms behind reproduction took many years to establish.

Around 1869, Friedrich Miescher discovered nucleic acids within cell nuclei. Decades later, Phoebus Levene found that nucleic acids were made of nucleotides, where each nucleotide was composed of one of four bases (called A, C, G, and T), a sugar molecule, and a phosphate group.

Further decades later, Erwin Chargaff found that in the giant molecule of deoxyribonucleic acid (DNA) that forms the payload of each chromosome, the quantities of the A and T bases are equal and the quantities of the G and C bases are equal, suggesting that A pairs with T and G pairs with C along the molecule. Chargaff explained all this to Francis Crick and James Watson on a visit to Cambridge in 1952.

Using Chargaff's rules, detailed X-ray crystallography data collected by Rosalind Franklin and Maurice Wilkins, and cardboard cutouts of the bases to figure out how they fit together, Crick and Watson discovered the double-helix structure of DNA in 1953. They celebrated their success by going to the local pub for lunch and announcing to the drinkers that they'd "discovered the secret of life."

What they'd realized was that the three-dimensional double helix structure suggests a mechanism for how DNA molecules replicate. During meiosis (part 1), the double helix unzips into two strands that separate base pairs into single bases, which then hook up with loose bases imported from the cell's liquid medium (the cytosol) until each strand forms a new double helix. In meiosis (part 2), these new DNA molecules unzip again to form the haploid gametes that seek their other halves elsewhere. This is the key to the biomolecular story of how genes are passed from generation to generation.

Four years passed until Crick stated the central dogma of molecular biology, namely, in brief, that the flow of genetic information in biology is from DNA to RNA to proteins. The dogma makes more sense when we add more details. In any (eukaryotic) cell, the DNA in the cell nucleus regularly unzips a sequence of genes to expose templates for the transcription of its information onto ribonucleic acid (RNA) molecules, which in turn act as blueprints for proteins. Molecular nanomachines called ribosomes crawl down the RNA strands, read the information coded in the base sequences, and interpret it as instructions to knit amino acids from the cytosol into long chains that build up protein molecules. As they're extruded, these macromolecules fold into predictable shapes that serve numerous specialized roles in the cellular economy.

In very large part, we're made of proteins. We contain tens of thousands of different proteins, out of the many trillions that could be assembled from the body's amino acids (and of the hundreds of millions the AI system AlphaFold has already mapped), and each protein has a unique structure dictated by those acids. Think of the proteins as specialized Lego bricks that our bodies assemble into all the microstructures and tiny machines that make up the contents of our trillions of cells.

All the recipes for cooking up this biochemical banquet are coded into the gigabyte or so of data stored as base sequences on the DNA molecule packed inside each one of the chromosomes within the nucleus of each and every cell in our bodies. The central dogma gives us a guiding thread to trace the story of life through all this dizzying detail.

The neo-Darwinian synthesis of evolution and the central dogma defines the modern science of biology.

FROM MUTATIONS TO MAMMALS

The myriad complexities that lie hidden in the modern story of life on Earth, as framed by the gospel of evolution and the central dogma, boggle human imagination. We can already see that huge gaps remain to be filled before we can sign off on the gospel and pronounce it complete. In fact, following our documented inability to pronounce classical mathematics or mechanics complete, we can safely bet that life – like logic and light – will find a brilliant way of escaping our understanding and forcing us to think anew.

One such gap is revealed if we ask what drives an organism to assemble itself and persist as a unified entity against all the surrounding threats that could damage or destroy it.

Evolutionary theory outlines an answer. Organisms have simply evolved with the driving imperative to live and breed. Any freaks that felt less driven would be weeded out of the gene pool without more ado. From the first protozoa onward, evolution on Earth has favored the success of species that fed and bred with more relentless efficiency. Even today, humans feel a powerful urge to grab resources and raise offspring at the most furious pace that the environment and competitors allow. Civilization has never tamed this primal urge.

The finer details would need to address the engineering of successful organisms from the Lego-brick possibilities offered by the proteins that our genes bequeath to us. Evolutionary variations have accumulated complexity steadily over billions of years, from primordial protocells to the higher mammals. How does increasing complexity help an organism to survive and thrive? Naïvely, we might imagine that a monoculture of greedy gray goo would have engulfed all in its path long ago, closing off the road to beings like us.

In a typical evolutionary just-so story, gray goo can't be as greedy as a modern mammal. True greed requires cunning and a sense of purpose, and gray goo has neither. Most life has no purpose. Life is a physical process that continues according to the laws of physics, including the law of rising entropy. Acting on purpose requires a sense of time, and time as we know it is an emergent conceptualization of change that means nothing to gray goo or other simple lifeforms. A life of purpose is a human thing, and it's our purpose-driven lives that make us the top predator on planet Earth.

Life's ever-increasing complexity over evolutionary time is an observed fact, but our explanation of it in terms of entropy is highly schematic. The incidence of the new mutations that drive the evolution of species is essentially random but occurs at a stochastic rate that we can use to calibrate a molecular clock, much as we use the random decay of unstable carbon isotopes as the basis for radio-carbon dating in archeology. The molecular changes that give rise to genetic mutations are triggered by quantum events, for which we can only make probabilistic predictions, not exact ones.

Random mutations in genes have driven enough variation to allow the evolution of eyes and brains and so on, even when typical mutations have negative or neutral effects on fitness.

Because living cells have efficient mechanisms for checking and correcting or compensating for the errors that emerge during the replication of DNA or the transcription of genetic information during protein synthesis, most of the mutations that quantum fizzing and popping throw up are caught before they can do any damage. But the lucky few benign mutations that make it past these mechanisms are enough, it seems, to drive the regular appearance of new species.

The statistics of all this defy the imagination. The problem resembles the popular challenge to Darwinists that likened evolution to a whirlwind in a junkyard and asked how a jumbo jet could possibly be assembled in such a chaotic way from junk. That problem was solved by breaking down the assembly into tiny steps, where each step was an incremental success paving the way for the next step. The steps are so small that the tree of life looks like an extremely slow-motion fountain. The march of the mutations is a quantum process, where the smallest possible mutation is a quantum transition in the state of a molecule. At the macrolevel of the story of life on Earth, this quantum granularity blurs out.

If the mutations that power evolution reduce to quantum state transitions, the logic of becoming can help us see how to open up the possibility space. Alternative possible mutations remain superposed with each other in a state of being before a quantum pop promotes one mutation into existence. In the superposition, they remain entangled with a fanout of other possible mutations in what may be extended rafts of entanglement that pop together into the occurrence of simultaneous mutations. For all we know, such parallelism could enhance the likelihood of lucky mutations in the history of life above the threshold suggested by the statistical challenge and hence solve the junkyard problem more decisively.

Returning to the question of what drives an organism to persist in living as a unified being despite all obstacles, we can appeal to politics at the cellular level. Life evolved from early single-celled loners (prokaryotes) to multicellular eukaryotes some half a billion years ago when cells solved the political problem of working together to raise their chances of survival by exploiting economies of scale, division of labor, and the sheer awesome power of being big. These advantages kept on increasing until they hit the physical limits on size in terrestrial gravity fields, as illustrated by the dinosaurs. This ceaseless struggle ensured that the genes required to become as big and greedy as possible were fixed in the gene pool.

All this may be fascinating, but we risk getting sidetracked from our primary mission, which is to build a new foundation for the science of mind and consciousness. Let's now take the evolution of human beings and their societies for granted by assuming that modern biology can explain it. Our next task is to review the biology of brains.

THE HUMAN BRAIN

Animals evolved brains to make their command-and-control networks more efficient by centralizing them. This process of encephalization was extreme in humans – the human brain is regularly touted as the most complex system in the known universe. We confidently regard it as the seat of consciousness and the mind. Here are the basic facts.

Human brains pack a lot into a small skull. Some hundred billion cells called neurons (counts vary), with about twenty billion big ones in the cerebral cortex (the two hemispheres) and many more smaller ones in the cerebellum (the so-called hindbrain), are the main actors. Each neuron has many thin projections called dendrites, which convey inputs from the sensory systems outside the brain and from other neurons inside the brain, and a long branching projection called an axon that conveys signals to other neurons or to the motor systems outside the brain. At the interfaces between neurons is a total in the brain of around a hundred trillion synapses, which are tiny junctions that can quickly be strengthened or weakened or made or broken (like overnight, during sleep) to remap the logic diagram of the cortex.

All this infrastructure forms the city map for an amazingly fluid and complicated flow of electrochemical traffic. When signals enter a neuron and raise its electrical potential from a resting level, the neuron gets more and more charged until it fires an electrical spike, also called an action potential, of some tens of millivolts that lasts for around a millisecond, either to other neurons or as motor output to the body. The neurons in an active human brain together fire up to hundreds of billions of spikes per second.

These nerve impulses travel at some twenty-five to forty meters per second, as Hermann von Helmholtz established in the nineteenth century. If you think this is slow, remember they're not photon currents in optical fibers but microscopic electrochemical tsunamis conveyed by the diffusion of ions. These miniature tidal waves can be enhanced or depressed by various neuromodulators, such as dopamine and serotonin, which act by influencing the ion flow at synapses.

We can model neural spikes to a first approximation as binary signals (0 or 1) in a layered digital network. The neurons that support or implement consciousness are in the cerebral cortex, a sheet about a fifth of a square meter in area and two or three millimeters thick, folded up to fit within the skull, where the neurons are stratified into six layers around big pyramidal neurons that branch within and between the layers. The cortex is quilted from modules, or minicolumns, about two hundred million of them, each a few tens of microns in diameter and containing about a hundred neurons.

Considered as a digital computer, the human cortex is impressively efficient, with a power consumption of around twenty watts. As of mid-2025, its capability when performing information-processing tasks can still sometimes exceed that of our latest artificial intelligence (AI) models running on our best AI chips in dedicated facilities that consume megawatts or more of electricity to function.

Neuroscientists and cognitive scientists often model the brain as a computer. For example, they say neuromodulators tweak the value weightings on the interconnections in its neural network. The computer metaphor hints at how we can refine the model to explain consciousness.

The hint is to look more closely at how the brain diverges from computers as we now build them. The key to the secret of how the brain implements consciousness is in the electrical traffic, which is rhythmic. Time plays a far more intimate and pervasive role in the way the brain processes information than it does in a computer with a classical "von Neumann" architecture, where a central clock imposes a rigid drumbeat that regiments the stepwise shuffling of data through the arithmetic logic unit (ALU) in the central processing unit (CPU). There the clock can run as fast or as slow as the hardware allows and can be interrupted at many points without wrecking program execution. Basically, computers don't have rhythm.

Brains process their data in a much more musical way. Our study of the finer points of this "music of the hemispheres" has barely begun. We can read out its basic rhythms and trace the main voices in the symphonies, but it's still hard to do.

Though we've studied brainwaves and correlated them with states of consciousness or unconsciousness for decades, most of this work is limited by the tech we've been using.

Electroencephalography (EEG) involves tracing electrical activity in the cortex using skullcaps studded with electrodes placed on the scalp. The traces have good temporal resolution (down to milliseconds) but poor spatial resolution, since each electrode records signals aggregating the activity of very many millions of neurons at once.

Magnetoencephalography (MEG) involves recording the magnetic fields around electrical activity between neurons and is comparable to EEG. The technique uses tiny and sensitive superconducting quantum interference devices (SQUIDs) to detect the magnetic fields, which are not blocked by the skull. The fields are extremely weak, but MEG spatial resolution for the cortical surface is down to millimeters, so MEG improves on EEG. But the traces still aggregate activity over millions of neurons – and they need some serious computing power for assembly into a useful map.

Functional magnetic resonance imaging (fMRI) is more complicated. It involves putting the brain in a strong magnetic field to line up the spins of the atomic nuclei in the tissue and superimposing a graded field to locate voxels, then using a pulsed radiofrequency field to cause nuclear resonances in the voxels in order to measure the concentrations of oxygen-rich and oxygen-poor blood surrounding the neurons. Oxygen burn indicates neural activity as the neurons fire, so a multi-voxel 3D map of oxygen hot spots gives a good indication of which brain regions are activated by the kind of activity going on in the subject's mind.

In the latest research scanners, a voxel in the 3D scan is a cube with side length of less than one millimeter. A full brain scan will contain several million voxels, yet even in the best case one voxel in the cortical surface still aggregates activity from many cortical minicolumns. A global brain scan can also show activity in the neural bulk below the cortex, including the bundles of fibers going down to the corpus callosum and the midbrain, with the hippocampus, thalamus, amygdala, and so on, but for recording brainwave activity the finest temporal resolution is a few hundred milliseconds, which leaves the temporal resolution of even the best fMRI scans far poorer than that of EEG traces.

Positron emission tomography (PET) scans produce results similar to fMRI scans. They involve having the subject ingest a radioactive tracer, which decays in the bloodstream to emit positrons that an external detector can record. As with fMRI, the detected events show where oxygen burn rate is high, so the technique doesn't measure thinking directly. Though PET scans are faster and clearer than clinical fMRI scans, the use of radioactive tracers is often a no-no.

The only ways we have to monitor neural traffic at finer resolutions are invasive – they involve inserting electrodes under the skull and risking damage to individual neurons. The technology here is still crude by brain standards, and we still have a long way to go, as two ongoing projects show.

The Neuralink approach is to use a special robot to implant a coin-sized device in a hole in the skull that connects to long, ultra-thin, flexible polymer threads that penetrate the cortex with micron precision. The threads carry a total of more than a thousand electrodes, whose signals can be recorded and interpreted in parallel. Neuralink's first human quadriplegic patient has been helped to "regain digital autonomy."

The Neuropixels approach is an international collaboration to make probes for brain research. The business end of one such probe is a thin silicon strip about a centimeter long with about a thousand microelectrodes arrayed densely along it, which is attached to an external interface and a box running the software. The system enables researchers to record from thousands of neurons at once with single-neuron resolution. It's now in regular use in animal studies and shows potential for human use in some surgeries.

The natural next step is to use AI systems to analyze data from such systems together with data collected in noninvasive studies and with anatomical images of neural circuitry to build up the full-brain circuit maps we call connectomes and hence working models of brain function.

We already have detailed functional simulations of generic cortical minicolumns as well as datasets for the connectomes of a few simple organisms. For example, the connectome of the fruit fly, with a brain the size of a poppy seed, maps more than a hundred thousand neurons and fifty million synapses and was published in October 2024.

Progress is accelerating in this area. In 2016, a project was announced to map the connectome of a tiny cube, no bigger than a grain of sand, of mouse cortex. In April 2025, the team published the functional connectome of a cubic millimeter of primary visual cortex in a mouse brain. This is functionally almost identical to a corresponding patch of visual cortex in humans, so we can learn a lot from it. The millimeter cube contains seventy-five thousand neurons and half a billion synapses, and its full dataset is between one and two petabytes in size. A similar dataset for the complete human connectome would be a few million times bigger.

There's more to report. In September 2025, a map was released of neural activity in a mouse brain while it decided how to react to an image. Seven years in the making, the study integrated the work of a dozen labs and recordings from more than a hundred mice fitted with helmets collecting data from hundreds of Neuropixel probes tracing over half a million neurons. The data, from brain regions that together covered just about all of the brain, showed neural activity first spiking in visual areas at the back of the brain, then spreading across the brain until the motor areas lit up. Once the mouse acted, widespread brain activity followed.

For our purposes, the outcome of all this work is that our knowledge of how the electrical spikes in the brain add up to consciousness is interesting but still meager. One big result is clear: The electrical traffic in the cortex as a whole is essential to consciousness. The rest of the brain also plays essential roles – the hippocampus helps lay down memories, the thalamus acts as a relay station for coordinating cortical traffic, the amygdala regulates emotionally-driven behavior, and so on – but the electrical music in the cortical hemispheres is really where it's at for consciousness.

The music of the intercourse between neurons is inaudible, but it's far richer than the acoustic music we make to entertain ourselves. Imagine a human brain as a city, with inhabitants corresponding to cortical minicolumns. Now imagine all those office workers busily generating digital data and sending it via wi-fi to other workers all around the city. The electrosmog above the city is a mess, so there the analogy breaks down, but imagine a science-fiction world where billions of cities over millions of years of fighting each other for dominance and survival had tuned all this electrosmog into symphonies of rhythmic vibrations in an evolutionary process, because each city's "vibescape" helped it in some crucial way to get ahead. That, roughly speaking, suggests how we might think of the music our brains play.

These vibescapes in and around human brains have been honed by evolution for life in the typical stone-age environments our hominid ancestors endured. But they haven't been honed to maximize the depth or the clarity of our insight into our own thoughts or to enable us to cut through to awareness of our ontic or epistemic predicament. Neuroscientists face an uphill battle in sorting out the scientific story here.

That story seems to be advancing toward the goal of consciousness. The vibescape in and around the brain is what our EEG and MEG recordings monitor and trace. Its symphonic waves spread over the cortex with vibrational frequencies in the dekahertz range (tens of hertz, which means tens of cycles per second). These vary from slow delta and theta waves, with frequencies from less than one hertz to seven or eight hertz, up to faster alpha, beta, and gamma waves, with frequences of up to eighty or a hundred hertz.

Among all these waves, we think forty-hertz gamma waves play a major role for conscious states. They seem be essential for the temporal binding that unites simultaneous but separate input signals into perceptions of structured objects. In logical terms, they build the structured sets that stand for rich real-world objects in our ontology. This forty-hertz dance of excited neurons binds them into groups that reinforce their neural interconnections – "neurons that fire together wire together." At a neuroscience meeting in Bremen in 1998, the neuroscientist Wolf Singer emphasized the importance of these forty-hertz waves for shaping the ideas that are central to consciousness as we know it. In the decades since then, this view has become established as the leading theory of how the brain performs object binding.

Within the brain, waves of neural firing go round in loops from neurons in the cerebral cortex to the thalamus and then back to the cortex. This thalamo-cortical looping continues for as long as it takes to structure and refine the thoughts carried by the waves up to the point where they emerge in consciousness and inform our behavior. The Colombian neuroscientist Rodolfo R. Llinás emphasizes the importance of this repeated thalamo-cortical looping for shaping coherent thoughts in his gloriously titled book *I of the Vortex*.

These thalamo-cortical loops link and couple oscillatory circuits in a mapping over the cortex that the Nobel laurate Gerald M. Edelman describes as forming re-entrant circuitry. He sees this as the key to conscious thoughts. Evolution by natural selection caused neurons to form neural groups that compete with their neighbors for relevance and resources in the cerebral economy within the body. The groups join into re-entrant circuits that link the cortex with the thalamus and other deep structures and thus give rise to the big waves we associate with consciousness.

Direct experimental support for this view has taken time. Chinese researchers said in April 2025 that they'd measured correlations between the strength and clarity of signals from thalamo-cortical loops and subjective reports of conscious awareness of stimuli triggering the loop signals. The subjects were medical patients who already had electrodes implanted deep in their brains to record thalamic signals, and they told the researchers when they were aware of threshold stimuli. When the team checked the cortical and thalamic signals, they found just the correlations we'd expect if strong loop signals were behind the patients' awareness of the stimuli.

But thalamo-cortical loops are only part of the story. The brain doesn't simply sit and wait for input to trigger neural activity. It's constantly churning with spontaneous activity as its neurons fire away, improvising looney tunes that make no sense. The rest of the brain works to organize these tunes into orchestrated melodies that are then trained by trial and error during interaction with the body and the outside world. This evolutionary process gradually ensures that they serve a useful purpose and make public sense.

The self-organizing activity has interesting dynamics. The brain works best when it stays in or near a critical state, where the individual neurons making up the network fire on average at a steady rate. Any deviations above or below this rate tend to push it toward either manic chaos or no neural firing at all. Neuromodulators tune this critical state, and accumulating evidence suggests that consciousness flourishes best when the brain stays near it.

But the brain also faces a range of evolutionary pressures. Sometimes it needs to act fast rather than carefully; at other times, a considered and customized response is favored over an impulsive reflex. To cope with this range, the cortex hosts two populations of neurons. One set acts fast but is inflexible and responds only rarely and slightly to evolutionary pressures. The other set is labile and easily retrained, which optimizes these neurons for deep and deliberative thinking. These two populations correspond respectively to the System One and System Two thinking styles discussed in Daniel Kahneman's popular book *Thinking, Fast and Slow*.

Generally, neurons fire together in rhythms that embody what we can call a language of thought. In this way, the time profiles of neural activity on many scales of frequency and duration seem certain to play a far more important role than timelines for a computer. During every waking moment, the brain hosts neural group raves of impressive vivacity. Yet still the synchronized spiking of neural groups seems a gulf away from consciousness as each of us knows it.

FROM MODELS TO MINDWORLDS

The cognitive philosophers Andy Clark and Anil Seth offer a useful account of the brain as a prediction machine. A mountain of work by cognitive scientists worldwide has contributed to their story of how mammalian brains can have evolved to provide the fine-tuned control and steering of our lives that we enjoy every day. This provides an experimentally established platform for our more ambitious attempt to grok consciousness in a wider theory of mind and being.

The brain has evolved to help intelligent organisms in their struggles to survive and thrive in our familiar world. It helps its owner to regulate the body and prepare it to do the right things at the right times. It takes sensory input from the world and builds an understanding of that world sufficient to make a life worth living from the materials at hand, all the while in collaboration with fellow organisms who share similar views and goals. To understand the world well enough to do all this is a tall order for a headful of noodles.

The neural network minimizes the cognitive challenge by maintaining a model of how it wants itself and the world to be in the future it extrapolates from what it knows or learns. In this way, it leads from the front by expecting the body to do what it can or must to keep up – to minimize deviations from goal states, in Clark's terms. Bodily appetites are enough to ensure that the brain stays focused on its goal states and updates its world model to stay ahead of the game. Recall Zen advice for athletes: Imagine the goal, and a well-trained body will follow. To lead a lifetime of achievement, the brain's job is to dangle a carrot that keeps the body moving.

Key to this conception is the world model the brain maintains. The parameters of this model are familiar quantities like pain, hunger, warmth, shelter, and so on. The model is an effective model – a heuristic kludge – which makes no claim to truth except in the evolutionary sense that it adapts its host body to its environmental niche.

Here culture intervenes. By trial and error, human societies have hit on the adaptive value of maintaining a shared world model and teaching citizens to conform to it. This shared or standard model doesn't cover personal details, naturally, but it provides a basic toolkit – language, basic science and tech, a few default moral and political perspectives – and lets the receiving brain customize or personalize the rest. In this way, citizens can largely outsource the cognitive load of refining and updating the model. The result is the scientific world with its standard models of cosmology and particle physics, as well as everything in between. Here we find a foundation model that we – here and now – are accustomed to regarding as the truth, if only in the weak sense that it's our best standard ontology until time passes and a better conception of reality relegates it to epistemology. The foundation model we've developed in science works well here and now, which is as much as we can expect from a product of evolution.

Together with David Chalmers, Clark developed another idea we can reuse here, that of the extended mind. The default view is to regard the mind as hosted in the brain, but the better view is that it's hosted in the world, which happens for the mind's owner to be centered on his or her brain. The novelty here is to be realist about how we view the world. We've all been hard-wired by evolution to view our own mindworld as the plain truth about the real world, and we need to exercise unusual epistemological sophistication to realize that most of what from within our own mindworld we think about reality is superficial or wrong. The extended mind hypothesis runs with realism and projects our mindworlds onto their physical footprint in reality.

Chalmers and Clark said that just as our brains and motor neurons are part of the way we find our way and impose our will upon the world, so our books, smartphones, satnavs, cars, work tools, and so on are parts too. If brains host minds, then so too do smartphones and so on, in the sense that they help us do what our minds make us do. The realm of mind covers all we do, out into the world.

Man the toolmaker is more than an evocative literary trope. Our minds embrace our tools, our domestic habitats, and our lived worlds. Mindworlds are us.

FROM GENES TO MEMES

This extension of mind parallels an extension of body in the evolutionary biology advocated by Richard Dawkins. In his first book, *The Extended Phenotype* (lightly glossed in his first bestseller, *The Selfish Gene*), Dawkins proposed that the human phenotype should be redefined to include our tools and technology, since they're essential parts of what defines us as a species. The standard version of the human phenotype is that it's a naked ape, as determined by our genotype, our genome. But that's a very reductive view, too reductive to explain what distinguishes us as a species (or a subspecies) among the apes, since it fails to reflect what's special about humans.

Pushing that reductionism further, we could say the naked ape is really just a mind – a cerebral music generator – hosted in a biomolecular robot serving as a survival suit to facilitate its life on planet Earth. We could say the whole DNA-based infrastructure of bodies and species and plants and animals that made us what are and keeps us alive is merely a wetware system for hosting the electrodynamic music that animates a mind, so let's forget about all that wet stuff, since all it does is help play and copy the music.

Dawkins' readers will recognize this reduction to a musical soul as a memetic variant of his selfish-gene idea: Instead of saying human bodies are mere robots for helping genes to replicate, we're saying human brains are just servers for helping the memes of our soul music to replicate.

In short, the whole shebang is essential to the way we do what we do, so we're well advised to keep it all in our thoughts when we ask how life and mind evolved on Earth.

SUMMARY

The endless story of the evolution of life and mind on Earth is complicated enough to demonstrate that the gulf between logical forms and physical systems on the one hand and living systems and consciousness on the other is enormous. It takes a multitude of facts – and a lot of hand-waving – to provide a satisfying account of how it all fits together. That story is the one we've briefly retold in this chapter.

Everything we know from biology strongly suggests that brains are the physical seats of consciousness. Neuroscience is the branch of biology that takes up the further story of mind. It's come a long way in a short time, but the new perspectives it has opened up are breathtaking, as we'll begin to see in the next chapter.

CHALMERS' HAIKU

Consciousness is odd;
neuroscience can't touch it;
loopy logic can.

MINDS

A special ceremony in New York City in June 2023, at the twenty-sixth meeting of the Association for the Scientific Study of Consciousness, starred neuroscientist Christof Koch conceding a bet he'd made twenty-five years earlier with his philosophical friend David Chalmers at the (smaller, second) ASSC meeting in Bremen in 1998. Onstage in New York, Koch presented Chalmers with a crate of fine wine.

The ASSC meeting in Bremen had as its theme the neural correlates of consciousness. As a young professor, Koch had worked with the DNA hero and Nobel laureate Francis Crick on their 1993 book *The Astonishing Hypothesis*, which described the neural anatomy and cognitive architecture of mammalian brains and focused on the visual cortex of cats and monkeys to argue that such work could well explain consciousness in the near future. Chalmers had recently published his big book *The Conscious Mind*, which argued that such work in the neurosciences would never succeed in solving the "hard problem" of consciousness. Koch bet Chalmers that it would be solved by 2023. It wasn't, of course.

To decide the outcome, competing teams of scientists had tested what by common consent were two leading theories of consciousness, namely the global workspace theory (GWT) developed by Bernard J. Baars and the integrated information theory (IIT) developed by Giulio Tononi. The experiments failed to confirm the predictions of either theory, so both the GWT and the IIT remained mere conjectures, and the hard problem remained unsolved.

To understand the issues involved here, we need to step back and review the larger context in which the ASSC drama took place. We've seen that consciousness is the big concept we need to grok in order to develop a theory of mind and a physical basis for the science of psychology. Ever since July 1990, when U.S. President George H. W. Bush dubbed the 1990s the "Decade of the Brain," there's been so much work on how the neural architecture of the brain could support consciousness that it takes more effort to keep the big picture in view than many insiders can summon.

Keeping that picture in view amounts to keeping a philosophical perspective. Psychology has been a favorite theme of thinkers in our philosophical tradition for millennia, so there's a treasure trove of wisdom on the mind to be mined. The challenge is to adorn ourselves with its gems judiciously, with due regard for their fatal seductiveness. Our aim is to find a helpful perspective on the science of mind, not to drown in spiritual wisdom, so we need to see philosophy as a trusted handmaiden rather than a divine oracle.

Our dip in the ocean of the philosophical tradition will be brisk and perfunctory. What we seek is a clean body of ideas to flesh out the facts we reviewed in the last chapter. Our quest is to find a way to advance from brain science to an understanding of how mind relates to the natural order.

FROM PANPSYCHISM TO ULTIMATE MIND

Let's start at the very beginning. Panpsychism is the ancient philosophical doctrine that the entire cosmos is conscious. According to panpsychists, everything that exists has a share in cosmic consciousness. The phenomenal world around us has a translucency that when lit from within by what Buddhists call the ground luminosity makes its determinate existence an illusion and its sheer being a gossamer portal into the unconditioned. With panpsychism, we can make glorious hay with the poetry of consciousness – as Annaka Harris does in her excellent little introductory book *Conscious* (2019) – but we get no help in developing a scientific theory of mind.

Panpsychists say the idea can explain how a property so radically unphysical at first blush as consciousness can have appeared in the evolutionary record, given that the primordial ingredients from which life evolved were not conscious. The doctrine thus shades into what we can call panprotopsychism: Everything can in principle participate in consciousness, but only brains like those in humans actually do so. Still, another new word leaves us short again. It doesn't either explain why brains became conscious or shine new light on the path to a physical theory of mind.

Colin McInnes proposed a defeatist view in his otherwise pleasing introductory book *The Mysterious Flame*. The "flame" of consciousness is as good a metaphor as we could want in this context. Mysterians accept that consciousness may be as inexplicable for us as astronomy is to a fish. McGinn says consciousness "supervenes" on the physical mechanisms in the brain that host it, which is to say it emerges in rather the same way that phenomenal properties like temperature and pressure emerge in the kinetic interaction of vast numbers of gas molecules. As many philosophers say, consciousness may be an epiphenomenon in the life of a working brain, just as a rainbow is epiphenomenal to a waterfall seen on a sunny day. The physical processes that govern a person's life leave no obvious causal role for consciousness in explaining why or how persons do what they do, and hence no handhold for an evolutionary explanation to refute epiphenomenalism.

A mysterian argument that deserves swift refutation is the idea that the self-referential loop involved in trying to understand the human mind from within the human mind is hopeless. As scientists, we cooperate globally to meld the best of our individual efforts into a single enterprise of superhuman power and reach. We're focusing a global infrastructure of minds and machines onto the challenge of understanding a single human mind, which is at root an animal mind. There's nothing hopelessly loopy about that quest. We're not like fish that don't do astrophysics.

Daniel C. Dennett was impatient with mysterianism and with the fuzzy mysticism of panpsychists and other promoters of naïve views about consciousness. He was a pragmatist with a predilection for cognitive science and evolutionary theory who liked to work with early robots and push the boundaries of primitive AI. He focused his critical scorn on the "qualia" that many philosophers defined as the phenomenal atoms of conscious experience, and said any attempt to interpose them between us and the physical world of naïve experience was otiose at best and probably meaningless.

Dennett claimed to have explained consciousness in 1991. He did so by describing in fascinating detail the various bugs and features of prevailing philosophical views of consciousness from the era before the Decade of the Brain. In his view, a judicious combination of the cognitive science of intelligent behavior and the evolutionary story of how the mammalian brain developed its capabilities should provide explanation enough to leave not only the mythology of qualia but also the whole folk psychology of mental states, intentions, emotions, and so on ripe for retirement.

For Dennett, even our human feeling that we're conscious may be just one more subjective illusion beside all the others that our species has cast off in recent centuries. Far from honoring consciousness with a physical place in the universe, Dennett said there's nothing to explain and we're chasing a bugaboo. But his fellow philosophers obstinately continued to agree with Thomas Nagel that science can never explain what it's like to be a bat, or me.

With his diagonal argument for the scientific inscrutability of consciousness, Chalmers is a central figure here. He's well aware of the charms of both mysterianism and panpsychism, but he's also close enough in outlook to neuroscientists and bullish technophiles to appreciate the strength of the case for scientific optimism. Consciousness is the crowning glory of the human mind, but Chalmers can see that neuroscience and AI tech will likely push our understanding of mind at least to within vanishing distance of Nagel's elusive feeling of what it's like to see red or be batty.

Here we recall our story of logical runoffs. Neuroscientists look set to advance ever further toward understanding how our brains host consciousness, until we find that resistance is futile in face of "conscious" androids. Our runoff logic can help us to map the science and tech that have already brought what Ray Kurzweil called the digital neocortex into existence. We can then consign our doubts to the realm of being, where mysterians can dig in. Lest this seem unduly dismissive, recall that any runoff tops out in being, so mounting a human last stand there is a good tactic. The realm of being is where ideas float free before the actions we've called pops crystalize out another layer of existence onto the ash-heap of history, and if a being can find a way to stay in being and avoid existence, well, that's one way to enjoy eternal life.

Before we dig deeper into the theory of mind, we need to clear the logical logjam here. Chalmers bases his case against the final victory of science on the inability of “third person” (3P) science to get a hold on the “first person” (1P) feeling of what it’s like to be oneself. He regards the 3P/1P contrast as so fundamental that it challenges the reach of science and makes a mystery of mind. But his argument puts more weight on the concept of a person than it can bear.

The logic of persons is clear enough. Persons are incarnated as composite beings, sloppy ones at that, which have no more enduring identity than Heraclitean rivers. They are extreme macrostates defined over complicated physical processes that are mostly peripheral to consciousness.

The enduring identity of a person is embodied in a macrostate that remains worlds away from exact definition. People are processes in spacetime that continually take in and push out surrounding stuff when they breathe and deal with food. In the world we still live in, people can normally be defined by their outermost spatiotemporal boundaries and by the traits they’ve acquired from either their genetic endowment or their habits and circumstances, but such limits are about to become fungible in the future world of medical nanotech, brain-vat cryotech, and mind-upload cybertech.

A person may be better understood as a legal fiction. Their ownership of personal consciousness may be advantageously regarded as a legal issue governed by property rights, political circumstances, or even religious traditions. As mystics might say, consciousness can’t actually be owned by anyone.

If persons are defined by questions of who owns what, the 3P/1P contrast has no role to play in a fundamental theory of mind. We might invoke the contrast to regulate interpersonal exchanges in a more practical and detailed psychology, but at the deeper level of philosophical foundations, it’s irrelevant. Consider an analogy: I don’t own the air in my garden, even though it seems distinct from the air in your garden, separated by the high fence between us. Analogously, I don't own the cloud of consciousness that my mind lives in just because it's separated from the cloud your mind lives in by two tangles of neurons, skulls, an air gap, and so on. We can insist that the 3P/1P story has no place in a theory of mind at the level of logic, math, physics, and philosophy.

An empirical test of this conclusion is readily available, in principle, given the present state of the art in neurosurgery. By cutting the corpus callosum nerve bundle that connects the two brain hemispheres under the skull, we can show that the hemispheres are capable of hosting their own largely separate centers of consciousness. Numerous studies have confirmed this, but the next step is new. If we cut the nerve bundles in two people and then splice a nanotech cable at one end to the nerves in the left hemisphere of person A and at the other end to the right hemisphere of person B, and then connect the other two hemispheres of A and B with another cable, we should create two new hybrid centers of consciousness. After an anticipated period of mental confusion in the two hybrid minds "Albert" (Al–Br) and "Blair" (Bl–Ar) as they learn to decipher the different neural codings in their newly paired hemispheres, we can then ask them how it feels to see red, be a new person, and so on. If they both respond that it feels much the same, we can take the conclusion to be corroborated: The 3P/1P contrast is a mere contingency that's of no great importance for a fundamental theory of mind.

Given that Chalmers' diagonal argument is based on that contrast, we could then agree that it fails. But it does succeed in showing that science as we know it will always seem to fall short on consciousness unless we change the default view or perspective most of us adopt on the nature of reality. That perspective relativizes our own views to our persons – who know what it's like to be ourselves, as Nagel would say – and relativizes the scientific view to an impersonal standard – "the view from nowhere," as Nagel did say.

We need to replace that default perspective with one that does justice to the well-nigh infinite malleability of personal views and their continuity with the celebrated objectivity of science. That objectivity is and must remain compatible with a corrigibility that can be dismally human in its shortcomings. There's an opportunity for a slippery slope here that collapses the 3P/1P contrast completely. Once this logjam has been cleared, the hard problem doesn't look so hard anymore. It's just another puzzle to solve on the way toward a sufficiently scientific theory of mind.

The unfinished tower of sets and the runoff story give us the logical tools we need to replace the old default view of science and the mind with something better. What we find is that human minds are limited reflections of ultimate mind. Logically, they're like the humble V-sets that reflect ultimate V. Ultimate mind is another face of ultimate V, an entity in being but not existence.

In conventional neuroscience, we take a 3P perspective on the brains of subjects whose 1P minds are always one step beyond the state of their neural networks. This gap invites some sort of cut – which looks like a ghost of the cut between classical and quantum reality made by Heisenberg and von Neumann, to general dismay. We can exorcize this ghost by adopting the radical perspective of ultimate mind.

In ultimate neuroscience, we take not "the view from nowhere" but the view corresponding to a V-set with a higher rank in the order of things than the humble rank of the V-set for the mind of the patient on the operating table. Shapes that shimmer in the realm of being for the mind of that patient are condensed as transient configurations of the electromagnetic field that buzzes over the patient's cortex. Being is reduced to determinate existence, and the patient's sense of "what it feels like" to suffer that state of being is reduced to the vibes from a neural spiking frenzy.

Such reduction of states of mind to states of the vibrating field of electromagnetic energy – carried by vast numbers of photons – over the cortex is somehow presupposed by much of what neuroscientists say about brains and mind. Many of their claims are philosophically debatable, naturally, but the group wisdom of the claims is unmistakable. We need only to accept that there's some kind of truth in the reduction and to tease out its implications in physics.

REVIEWING THE CLASH IN NEW YORK

We can make progress toward a science of consciousness by reviewing the two theories we mentioned that failed to save Christof Koch the shame of buying wine for Dave Chalmers. Let's start with global workspace theory, also known as global neuronal workspace theory.

GWT has been in discussion for some decades and counts as a mature theory in the tradition of cognitive functionalism, which takes the computer analogy for the brain seriously and draws the consequences. Bernie Baars, its author, is impressed by the fact that computers with von Neumann architectures have a central workspace in and around their CPUs and feels that the brain must have something similar. A computer's central ALU is surrounded by caches and registers that hold the program and the data that the machine is currently running and processing, so the brain must have a central workspace, perhaps in the frontal lobes of the cortex, where the neural activity currently supporting consciousness is running. In the functional terms that cognitive scientists esteem, this is an eminently logical idea to pursue.

Against GWT, a fundamentalist about consciousness can insist that it leaves the real mystery untouched. Neural activity in the workspace may correlate well with consciousness, but the hard problem of how a buzz in the brain can feel like me is totally ignored.

Dan Dennett had a lot to say about theories like GWT. His basic line was that they offer all we can expect of a scientific theory of mind. If we fill out the story in enough detail – as we're doing with petabyte connectome mappings and the like – the time will come when people no longer complain that some deep mystery is being ignored, any more than they complain that our theories of weather and water ignore the fact that we get wet in the rain. Instead they'll see the residual facts left unexplained by a cognitive theory of mind as brute facts of nature, period. People used to complain that biology didn't explain life, but today, with artificial life on our lab benches, they don't. It'll be similar with consciousness.

Fundamentalist complaints do have something to add, and this is something Dennett felt strongly about. A global workspace in the brain is like a theater – Baars has made much of this simile – and thus begs the question of who or what the audience is for the productions staged in it. Dennett scorned the idea that homunculi in the audience enjoyed the show by asking whether they hosted tiny crowds of smaller homunculi in their homuncular workspaces, and so on, inviting a nightmare of homunculi all the way down. Such an infinite regress of mini-minds is just the sort of problem for 3P science that Chalmers' diagonal argument exposed.

Dennett's way out of this predicament was to embrace the homunculi. Functionally speaking, the brain was packed with little cognitive robots that worked much like the nightmare homunculi to serve as audiences for each other's work. The idea was that these robots competed for "fame in the brain" by recruiting their audiences into movements big enough to surface in conscious behavior. These robots were arranged hierarchically into ranks and got simpler the further down the stack they were, down to on-off switches at the bottom. As Dennett once said: "It's robots all the way down."

In summary, GWT is based on a computer simile that has limited utility for explaining how consciousness arises. It can explain the functionality but not the phenomenology. Brains aren't computers, so the GWT architecture model is too static, with no ear for the music of the mind.

The second contender in the 2023 showdown, integrated information theory, is a more recent and more sophisticated attempt to square the circle. IIT's main proponents are Koch and Tononi, whose passionate advocacy of its merits serves to distract from the conceptual and mathematical obscurities in its current formulation.

As currently formulated, IIT is defined by five axioms and five postulates, all expressed in technical language that needs to be unpacked carefully to make sense. The theory leads up to a mathematical quantity called Phi that its proponents say can measure the consciousness of a system, such as a human being or another animal, or indeed a digital machine. Tononi is a psychiatrist as well as a theorist, and his colleagues have used Phi to "measure" the consciousness of patients in coma and other clinical states.

IIT begins with subjective experience, which it defines as structured sets of distinct phenomena that form qualia. The definitions aim to regiment sets of qualia in such a way that they're correlated with specific configurations of neurons embedded in causal networks. The Phi of a state of the brain is then defined via the complexity of these neural configurations by quantifying the "causal power" of the networks to change experience, either by reshaping the network from within or by causing effects in the world external to the subject. Roughly, the more information the system can reflect within itself and the more integrated the information becomes thanks to the internal connectivity of the network, the higher Phi is, and the more conscious the subject is deemed to be.

Philosophically, all this is thoroughly debatable. Four lines of criticism spring to mind. Let's check them in turn.

The first is the issue Dennett would start with. Invoking qualia to define subjective experience revives questions that have been rehearsed ever since Edmund Husserl sought to establish phenomenology as the foundation of psychology in the nineteenth century. The strategy of attempting to define a layer of subjective experience like this between us and the objective world beyond has been tried many times and has never quite worked out. The phenomenological language that results from the effort seems fuzzy, obscure, and redundant, and a skeptic can simply ask: Why not cut out the middleman and go straight for the real world? We don't see qualia; we see things out there in the world.

The second issue has an even longer back story. Invocation of causal networks recalls questions that have been rehearsed ever since David Hume aired skeptical doubts about causality in the eighteenth century. These doubts were ignored rather than answered by Judea Pearl's recent efforts to buttress the math of causality by defining counterfactual models, which only fit the bill if we're prepared to be more generous about entertaining counterfactual scenarios than many scientists can easily accept. Invoking "causal powers" may help us see how cognition works, but it can't help us with experience.

Thirdly, the conflation of consciousness with complexity misses the point. Complexity correlates with intelligence, and intelligence is not consciousness, as we know from the latest AI chatbots. There is a link of sorts here – it's hard to imagine what a conscious system would look like if it were too simple to evince typically conscious behavior – but that seems more like a limitation on our imagination than an essential requirement for phenomenal consciousness. Panpsychists and epiphenomenalists would certainly say so.

Finally, at the core of the IIT story is the correlation of carefully defined configurations of qualia with configurations of neurons embedded in somehow defined causal networks. We have an elaborate theory of the neural correlates of consciousness that lacks any indication of why the complexity or causal power of those neurons should result in something so remarkable as a phenomenal world.

We conclude that IIT is no more the final theory of consciousness than GWT was. It shows promise as a pragmatic theory for use in a clinical and diagnostic context, but there's no deep new science behind it. We need to move on from the 2023 showdown in New York and look further afield.

INTENTIONAL MODELS

As we've seen, effective models serve us well as stand-ins for the world, so much so that we find it almost impossible not to regard our personal world models as identical to the big world of public life. These models are sustained by integrated neural activity and have causal power to shape our behavior. This is the philosophical side of the truth we need to develop if we seek to move on from IIT toward a theory of mind. We each have different views of the world, and these differences are ineradicable, if only because each of us sees the world from a personal perspective that quickly fuzzes or blanks out into ignorance or indifference. Our own personal worlds have smaller and simpler models (mapping to V-sets in logic) than the model (or rather time series of models) corresponding to the shared world of science and public life.

Let's call these personal takes on the big world of science mindworlds. And let's call the big world of science and public life the real world – with the proviso that it's not the ultimate world beyond all phenomena but just the latest and best take on the ultimate world that science and taxpayers can deliver. Like mindworlds, the real world thus understood is mapped logically to a V-set, with the difference that it's a much bigger V-set than that of a typical mindworld.

Recalling Thomas Nagel's views for a moment, this take on the real world is emphatically not a view from nowhere. It's just a view from a higher or more privileged perspective than that available to the average person, who by contrast is stuck inside a mindworld with very limited horizons. Naturally, we all aspire to live and work in the real world, and we certainly intend that what we say or do has an effect in the real world, but what it feels like to be me is somehow lost in the wash as I swim up through the ocean of being to reach the surface of the real world, where my mindworld is one among billions in the murky depths. Extrapolating, we may surmise that our vaunted real world would be lost in the depths if we could fly upward through the ether to the ultimate world where our logic is lost in a psychedelic paradise. This suggests that what a world feels like is defined by its limits.

We regard any world we inhabit as real. My mindworld and the real world of science compete for the honorific “real” in my consciousness. They do so in a runoff that repeatedly pits ontology against epistemology along a time dimension. This runoff escapes what would otherwise be the flat contradiction we’d face every time reality surprised us.

The philosophical jargon for conflating symbol and reality as we do at every step in the runoff is to say that our attitudes to the models are intentional states. Such intentional states are psychological fudges: We conjure up mental images that we take to be symbolically – that is to say, magically – connected to what we intend them to be. This magical thinking implies that the jargon is lipstick on a pig.

The idea that I intend something to be so, and therefore it is so, is not merely magical but mad. Yet in psychology such mental states are commonplace, so philosophers have had no option but to go along with the madness and simply label it wherever it arises. For example, qualia are what we say they are – and that’s just the problem with them, because it means we can’t get an objective handle on them.

New variants of the intentionality problem arise constantly when we attempt to build the theories of meaning we need to explain how we can “mean” anything at all with what we say. How can a modulated puff of hot air from the mouth or a few squiggles on paper mean anything at all? A language is a game we all play (as Wittgenstein said), and those who understand our utterances simply take us by convention or tradition to mean what we say – perhaps in a Tarskian way as saying what they too would say with the same words. This gameplay also applies to our own mindworlds of meaning: We simply play the game as if our mindworlds were real. Playing the game is what counts if you’re not a philosopher.

THE MIND IN THE MIRROR

A mindworld is a part of the environment of a human being. Its granularity, in terms of its formal structure and of its user interface, defines it as a macrostate of the physical environment. Because its formal structure is determined by the ranks of the sets in the stack that models the cognitive powers of its owner and its user interface is dictated by its owner's sense organs, such a mindworld is a personal possession, linked only by intention to the real world.

The macrostate in the mind is identical with a macrostate of the human ape, or at least intentionally identical, which is to say the ape becomes dimly conscious of the real world itself. The interactive embedding of the ape in its world ensures that this consciousness dawns as awareness of a classical landscape, where the ape's irrepressible habit of stomping on quantum bubbles suppresses quantum fizzing and popping to below its cognitive threshold.

We can invoke a useful metaphor here. A cognitive mirror reflects the mindworld of stuff experienced by the ape. The mirror hovers – imagine a mirror ball in a disco – in a state of being for the ape, who sees the image in it but not the mirror ball itself, yet in principle this ball of being remains accessible as an object for any outside observer who studies the events within that ape's brain.

The mirror ball is a logical metaphor. The mirror reflects not only a visual surface but an entire structural breakdown in terms of rational categories as well as the perceived sensory properties of a multimodal image. Logically, it maps to the V-set poised in a state of being as a proper class for a theory about sets of lesser rank, or the view from below of the V-set reflecting the logic of the ape's mindworld. The mirror ball metaphor is a way of positing the existence of something physical to reflect that mindworld.

The mirror image reflects the macrostate of the world that matches the ape subject's cognitive capabilities, which in turn match the complexity of the stack of forms we could invoke to model those capabilities. The mirror respects the subject's cognitive limits by reflecting only what the brain can process. It reflects the conscious subjectivity of its owner.

But where can we find this magic mirror ball in the brain? Is it perhaps in the pineal gland, which René Descartes guessed was the seat of the soul? No, it's not that easy.

First, recall that it's a logical mirror. Its job, in evolutionary terms, is to reflect a world model that helps the agent hosting it to flourish by dangling a carrot to keep the agent moving. The mirror reflects a world model. A conscious agent is aware of that world model as the reality in which it (or he or she) is embedded, just like a person using a bathroom mirror is aware of the mirror image as (a true reflection of) their own face. As the philosopher Thomas Metzinger said in his popular book *The Ego Tunnel* when explaining something very like this setup, we're hard-wired to see the model as reality: It's like a VR scenario that we can't help but regard as real.

We've seen that the model is a predictive model. The brain takes time to process its inputs and get its thoughts together, so to avoid being permanently belated with its prompts for action, it extrapolates from what it knows and postdates its model to make it predictive, which makes it work as a carrot, an inducement for the body to persevere as it finds ways to interact fittingly with the world. The predictive model also helps explain the neuroscientist Benjamin Libet's puzzle about how the brain manages to orchestrate real-time action when its neural processing is so often belated.

We can begin to bring the story into line with physics here. The world model is supported by rhythmic neural excitations in the cortex. These excitations generate a pulsating electromagnetic (EM) field within and surrounding the brain. The frequencies of the notes and melodies in this symphonic field range from the gigahertz cloudbursts of spikes from billions of individual neurons to the slower kilohertz, hectohertz, and dekahertz waves that roll over the entire cortex to create the larger symphonic harmonies in the music of the hemispheres. Consciousness is symphonic: Any music buff will tell you that the best music takes over their consciousness and leaves no first-person "I" hanging back from the enraptured me.

Like all EM fields, this cerebral EM field is in quantum terms a cloud of photons. The photons in and around the brain have long wavelengths (many kilometers in free space), so they're not strictly confined in the skull, but they're tiny. Hectohertz and dekahertz photons have energies on the order of decillionths of a joule (that's less than a quadrillionth of an electronvolt). Individually, such photons are drowned in the thermal noise of a living brain. But collectively they embody the bass notes of the tune that leads an organism on its merry dance through life.

Some bold researchers have dared to speculate about how this cerebral EM field gives rise to consciousness. One such is the molecular biologist Johnjoe McFadden, who proposed in 2002 that the EM field created by the brain's waves formed a "conscious electromagnetic information" (CEMI) field. This CEMI field became the centerpiece in his theory of how the brain made the mind. The theory relied upon an unripe idea of how patterns of information could give rise to phenomenal experience, so it suffers the same vulnerability to criticism as integrated information theory (IIT) in that it provides no link between information and experience. But a remnant of the idea remained in the book McFadden co-wrote with British TV celebrity physicist Jim Al-Khalili on the brave new world of quantum biology in 2014.

An earlier example of a much bolder speculation about the cerebral EM field was the 1990 suggestion by popular author Danah Zohar and psychiatrist Ian Marshall that some of the photons in the brain form what we call a Bose–Einstein condensate (BEC), which is a state of matter that typically arises when extremely cold atoms behave like bosons (because they have an even number of fermions in them, with spins adding up to an integer) and so fall into a single coherent quantum state. These ultracold bosons form an extended condensate that acts like a giant quantum object. The idea behind this bold speculation is that it's the only apparent way of physically uniting disparate microscopic parts into a single entity. But neither Zohar nor Marshall said enough in their book about BEC states to make their idea seem remotely feasible.

Despite the seeming absurdity of the idea, the cerebral EM field may in a relevant way resemble laser beams, which are often seen as examples of BEC states. To see why, recall that "laser" is an acronym for "light amplification by stimulated emission of radiation."

A laser is essentially a box or a cylinder with a mirror at each end that contains a lasing medium. The medium consist of atoms in excited states that we can cause to fall back to states of lower energy by "stimulating" them with photons of light that has the same frequency as the light the atoms emit when they're stimulated. In the box, large numbers of these monochromatic photons ping-pong back and forth between the mirrors until they line up with each other in a coherent state. If one of the mirrors is half-silvered, some of the photons escape from the cavity as a coherent beam for as long as we "pump" the laser with energy to keep restoring the atoms to their excited state.

In the magic mirror metaphor, thalamo-cortical looping pumps coherent states formed by the dekahertz photons of the cerebral EM field. These states don't persist, as in a laser, because the laser analogy fails here. The warm and churning tissue of the brain is a thoroughly hostile environment for coherent states. But still the photons momentarily team up with each other to form tiny pockets of coherence in the big waves we observe in EEG traces. That's how photons behave, even when they're jostled amid crowds of other photons with higher or lower wavelengths. Even sunlight, which was Max Planck's prime example of incoherent thermal radiation, shows rainbow sheens or little bursts of speckle as it reflects off surfaces, all of which result from coherent interference of photons with the same wavelengths.

Recall the metaphor of the flame reflected in the mirror. A flame is a classic source of incoherent thermal radiation, so it can't be a source of extended coherent states. But we don't need such extended states to model a mindworld, because, as we deduced from the runoff story, a mindworld is a transient phenomenon, ready to be replaced in a fraction of a second. We're only looking for a physically feasible description of the 1P experience of what it's like to feel this constant turnover of moments. We're not looking for an eternal soul.

In New York, at the 2002 conference "The Self: From Soul to Brain," California theologian Nancey Murphy expressed the conventional view that the soul is the form of the body, but then she freshened up her tale by opining that this was an information structure and that its immortality was comparable to that of any such structure, or indeed of any mathematical entity, which made it distinct from the brain. In our tale, the photon dance within the flame of consciousness provides a point of anchorage for the soul in the obvious sense that it instantiates some mathematical structure. This lets us say that when the flame is snuffed out, the soul departs to its eternal resting place in Plato's heaven.

The flame of consciousness emits a flickering glow of tiny and momentary bundles of coherent photons. Collectively, these convey a continuing stream of information about the transient states of the cerebral EM field that carries the music of the mind. When the flame is gone, the music stops, and vice versa, in a reciprocal dependency we know for sure is more than mere correlation. This identity leaves us pondering the mysterious role of time. The flame of consciousness plays an ineffable melody, and the music is temporal.

The idea that the personal soul incarnated in the brain is carried by coherent BEC states of the photons in the cerebral EM field is wrong. The EM field doesn't project a hologram of the subject like the image of Princess Leia in a *Star Wars* movie. Cerebral photons don't maintain their coherence for anything like long enough to get their act together as the laser beam for such a holographic light show. Nevertheless, they do magnify one particular photon property just enough for it to register in consciousness.

This property is the curious one remarked upon earlier that in relativistic spacetime a photon ages by zero proper time between emission and absorption – as Einstein said, time stands still for a photon. The first and most salient feature of consciousness is that it registers the passage of time. Time flows by in a stream composed of droplets. The droplets are moments of specious present – droplets of "now" – each of which endures for a fraction of a second, typically for around a hundred milliseconds, a duration close to what's called the flicker fusion frequency. This duration is slightly longer than the inverse of the screen refresh rate of a phone or computer or television, or the frame rate for a traditional movie, where our visual system smooths out the flickering to let us see continuous flowing motion.

The duration of the specious present corresponds handily, as a quick numerical check can confirm, to the Heisenberg uncertainty in the temporal location of dekahertz photons.

Quantum jitters blur the evanescent moments of eternity these photons carry in the cerebral EM field into what we conscious beings experience as the flow of moments that mark the passage of our time on Earth. If those photons cohered into more prolonged BEC states, they'd extend our "now" moments, so we can infer that the laser action in the magic mirror is nonexistent. The photons can only cohere for small multiples of their Heisenberg uncertainty.

Conversely, if the human flicker fusion frequency were higher, we'd expect the frequency of the brainwaves behind our consciousness to be higher too. Since many birds have higher flicker frequencies than humans do, they may have higher-frequency brainwaves in their alert state, as well as briefer "now" moments and a perkier consciousness. This is a potentially testable hypothesis.

Consciousness is temporal, but the soul is eternal. The mind is physical, but the soul is mathematical. We can now forget about souls and return to terrestrial minds.

Another feature of the photons in our cerebral EM fields that may be significant is their wavelength – which again, by Heisenberg uncertainty, corresponds to a fuzziness in their spatiotemporal location. In this case, uncertainty smears them out over thousands of kilometers – as big as planet Earth.

Despite the rapidly diminishing probability of such ultralong waves being detectable far from the brain, this feature lets us feel that deep down we are all united in a global mind.

In short, a world model is coded in pieces of soul music carried by the flame of photons that together comprise the cerebral EM field. The subjective side of the world model is a mindworld, a world of consciousness. This is what our mental VR system gives us. It's the topic of our final chapter.

SUMMARY

Consciousness is still a mysterious feature of life on Earth. We've explored how our logic of being and runoffs suggests how we might penetrate the mystery and see how to resolve it in science via the collective behavior of photons. This led us to propose a novel but physically motivated hypothesis to explain the temporality of conscious experience.

The history and philosophy we discussed along the way offered some welcome context and mood music, but the main outcome stands clear of all that. Logic and physics are enough to explain the temporality of consciousness, and the problems we mentioned about what personal mindworlds feel like or how intentionality works for our mindworlds are best left as puzzles for philosophers.

We shall propose nine fundamental laws of psychophysics in the next chapter. They offer a summary outline of the logic and physics we need for a grounded view of psychology.

NEURATH’S HAIKU

At sea in a boat,
we rebuild it plank by plank,
trying not to drown.

WORLDS

The founder of psychology, William James, used the phrase "worlds of consciousness" in his lectures published in 1902 as *The Varieties of Religious Experience*.

For James, consciousness was a stream, where the droplets forming the stream were lost to our awareness within the continuity of its passage. The stream flowed through us to carry moments of present awareness into the past.

Jamesian consciousness took the form of transient worlds. These were subjective and corresponded only superficially to the objective world we all assume we live in.

James' worlds of consciousness are our mindworlds, or so we can claim. They're small and blurred and embody only the phenomenal world reflected in the subject's cognitive mirror. Also, they're centered in a way that demands recognition in any theory of consciousness.

The creative center of a mindworld – its fountainhead – is something we can dub the omphalos, where "omphalos" is a Greek word (like "ouroboros"), which means navel, with the related meaning that's relevant here of "navel of the world." (For the ancient Greeks, the omphalos of the world was the temple of Apollo in Delphi.) A natural alternative way for us to refer to the omphalos of a mindworld would be to use the unfortunately overused word "I," but doing so too freely here would merely invite confusion. A mindworld always has an omphalos, which the subject or the self of that mindworld would normally refer to as the "I" of the vortex or ego tunnel through which that mindworld grows into being.

The perimeter of a mindworld depends on the cognitive reach of the subject for whom (or which) it bounds the world. But it always seems infinitely remote, because the existing furniture of a world shades off into being and then nothing toward the boundary. The world is everything for its subject, who (or which) has no conception of what lies beyond. That's what makes it a world – and what made the cognitive mirror metaphor necessary in the last chapter. The mirror was what made the mindworld of someone in a brain scanner pop into existence for the operator making the scan.

As logical guidelines for the discussion of mindworlds, we do best here to recall the formalism we reviewed earlier in our conspectus on set theory.

Let's present the first idea we propose to highlight for later reference as the first law of psychophysics: Subject and object are equal and opposite.

Like sets, which have two sides and appear as classes from below and members from above, everything around us – everything in our ontology – is two-faced: It's an object from above and a subject from below. Where we stand when we interact with it is what makes the difference. As sovereign beings, we prefer to look down on things as objects, but in some cases that's impossible. We have two main special cases. The omphalos, the origin of the world, is like the empty set: It's a null object, with no accessible interior. And the mindworld itself is like V, the universal class. It's a pure subject, whose objectivity is inaccessible from within. A vivid way to understand this is that beyond it is the ouroboros, the loop that takes us right back to the omphalos.

The second law of psychophysics can be seen as a pendant to the first and goes like this: Subjects have being whereas objects exist.

Set theory shows us that being and existence almost always go together. Generally, sets seen from below have being, but seen from above they have existence. Applying this duality to the typical furniture of a human mindworld may seem wild – what kind of being do tables and chairs have? But wild or not, it's possible. Chairs aren't too intelligent or integrated – they generally just persist in being what they are, and they fall apart rather easily, so they're not conscious – but they do offer a world of experimental possibilities for any condensed-matter physicist who chooses to study them.

Notice the new move here: One subject can enter into the world of another. Human subjectivity, at least, is surprisingly polymorphous. We can, albeit in a limited way and mostly in imagination, adopt the shape of another world, such as the world of model aircraft or that of an alien planet in a science-fiction movie, or indeed that of chairs. More interestingly, we can enter each other's worlds, as we do every day in our family and social lives. Such incursions are as superficial and fuzzy relative to the target world as all individual human worlds are relative to the real world of scientific investigation, but there's no way we can entertain a plurality of worlds without allowing for some such means of access.

Incursions between worlds are the price that logic demands of us for a plurality of worlds. As the omphalos of my world, I can't conceive of any worlds other than my own unless I can somehow adopt their form and breathe life into them. In the logic of becoming, the creative self is a polymorphous entity that can deploy its morphing powers to adopt a personal or a scientific or another perspective. But it will always be locked into a bipolar perspective, with a proximal "here" pole and a distal "there" pole. If the proximal pole is my everyday human subjectivity, the distal pole may be the world of model aircraft or sci-fi movies or whatever, which I can enter so deeply that the poles seem to flip or trade places: I can be so absorbed in a movie that my everyday world sinks back into existence, waiting for the spell to lapse and dump me back into a new iteration of the present, sitting in my armchair. Even when I contemplate the early universe, say, musing on the glory of its unfolding, my omphalos is breathing life into the subject and enjoying ecstatic union with the cosmos until the enthrallment passes and I come back to my senses.

The third law of psychophysics introduces something we met in set theory and quantum physics and now assign a wider significance: Subjects become objects via ontogenesis.

Ontogenesis is what makes me an object as soon as a drop of time has passed by, borne along by the relentless river of fleeting moments, the same river we met earlier that bathed our Heraclitean personal identities in the chill of a vanishing eternity. Our mindworlds don't persist for long. My being at this moment is soon the pale existence of a fading trace in the neural correlate of my memory, plus a cooling impression on the chair and a drying residue in the coffee cup.

The first three laws of psychophysics define a logical frame within which we can go to work. They don't come anywhere near to being a theory of consciousness, but they do give us a crude first idea of what the worlds are that populate this new metaphysics for a physical theory of mind.

The logic of worlds has a long and rich history in academia. Let's take the time to review it more carefully.

THE LOGIC OF WORLDS

Statements can be true in this world or true in all worlds. The logic of statements that are necessarily true or false, or possibly true or false, also called statements with modalities or modes, began already before Aristotle. But in its modern and more formal incarnation, modal logic began with the work of C. I. Lewis, who between 1912 and 1932 defined a suite of axiom systems S1 to S5 for the modal operators *Nec* and *Pos*, where *Nec* P means necessarily P and *Pos* P means possibly P, for any statement or proposition P.

We understand the relation between modal systems and worlds like this: A statement is necessarily true if it's true in all possible worlds and possibly true if it's true in some possible world. Pursuing this possible-worlds approach, Saul Kripke published a completeness theorem for the Lewis systems of modal logic in 1959, while he was still a teenager. Apart from the worlds, his proof followed the lines of the completeness theorems for classical logic.

More technically, a Kripke frame is a set of worlds together with a binary relation over those worlds to say which worlds are accessible from which others. The frame defines a tree of nodes, where the root node is the actual world. And a Kripke model is a Kripke frame together with a satisfaction relation specifying which modal formulas are satisfied (or true) in all worlds (a formula with an open x in it is satisfied in a world when the satisfaction relation maps an object to x that makes the formula true, and an x is open when it's not bound by a quantifier). The rest of the model theory is standard.

Kripke's semantics for modal logic triggered a flurry of activity among logicians that lasted for decades and produced a host of new systems, proofs, and philosophical debates. One notable result was based on the earlier finding by Kurt Gödel that intuitionist logic was equivalent to the Lewis system S4 under the mapping of the intuitionist assertion P to the S4 assertion *Nec* P. Given that finding, Kripke found it easy to prove the completeness of intuitionist logic by reinterpreting his possible-world semantics.

For philosophers, such possible worlds allow a variety of interpretations. They can represent epistemic worlds, as they do in intuitionist or constructive logic, or logically possible worlds that differ from the real world in the contingent facts they contain, as in counterfactual worlds, where anything that could have happened is true in some counterfactual world or worlds. Or the variety of possible or counterfactual worlds can be restricted in some way. For example, we can insist they be physically possible relative to some theory, such as classical or quantum mechanics or string theory (this undefined freedom of content for counterfactual worlds is why we said in the last chapter that Judea Pearl's attempt to define causality in terms of counterfactuals wasn't the final answer to Hume). Outside physics, philosophers exploring and explaining the semantics of natural languages have invoked counterfactual worlds to define truth and meaning for a wide variety of everyday expressions and turns of phrase.

What all this technical work implies for us here is that the new systems of modal logic, constructive logic, temporal logic, and other kinds of logic have begun to sidestep the problem we noted earlier that classical logic is too "flat" to account for the logic of change.

We took the lesson from this flatness problem that the set theory of the cumulative hierarchy was a useful formal metaphor to help us develop a deeper appreciation of the logic of time and change. The reason for holding back at the level of metaphor is that we can probably build a better theory of change in a new theory of worlds. In future, logicians will use the new systems of constructive logic interpreted in terms of pluralities of worlds to try to do so.

Set theory was developed in classical logic. That fact led to two big limitations on the concept of a set – almost absurdly general though it might have seemed when we first reviewed it. The first limitation is that sets always have a definite size, or cardinality. This is obviously impractical outside pure math – who can say exactly how many chairs there are in the world? The second limitation is that sets A and B are either identical or different, with no shades of similarity in between.

We can escape both of these overly restrictive assumptions by moving on to category theory. Categories don't need to have fixed cardinalities, and they're said to be isomorphic (or not) to each other, so strict identity is unnecessary. This gives us a much looser and more flexible formalism. Very crudely, category theory is what you get when you try to develop set theory in a constructive logic. But it's a more recent theory, with extremely little so far in the way of digestible secondary literature, so it seems unwise to try to dig deeper into category theory here.

A new logic of worlds is likely the right course to pursue. But it would be out of place to embark on a formal reckoning with such a logic in this book. We're reviewing introductory psychophysics, not getting down and dirty with the technical systems for handling possible worlds.

THE PHYSICS OF WORLDS

Physics applies to the real world, but it's not clear how far it applies to mindworlds in general. We can build fictional or fantasy worlds in which this or that physical law is suspended, and even for the real world we can discover that the reality surrounding us deviates in some way from what we imagined to be the applicable physical laws – that's how we create new physics, after all. But what we can't do with any mindworld is imagine it free from any physics at all.

We can't do so for a simple reason: Mindworlds, like our minds, are subject to time. Each momentary mindworld has a timestamp that dooms it to fall into existence as the logic of becoming moves on. And once we admit time, we admit space – or at least we do if we accept the relativistic equivalence of the spacetime dimensions that our recognition of photons as the stuff of both minds and worlds commits us to. Without photons, our mindworlds evaporate, and with photons comes a lot more physics. Following through consistently, we may even find we can reconstruct much of physics as we know it, right up to its two standard models.

But what about math? The mathematical worlds of sets or numbers or bits seem to float free of physics. Well, "seem" is the operative word here. Abstracting from photons, the least we need to make a mindworld at all is a supply of bits to define it, or rather a supply of qubits, which pop into bits in our logic of becoming, and this again commits us to physics.

Information is physical, said the IBM Fellow Rolf Landauer in a 1991 paper citing Shannon's account of entropy in terms of information theory. A philosopher or a mathematician or a fantasist can invent an apparently self-contained world of abstractions or fictions without reference to bits, but we in the outside world must still unpack it bit by bit.

In our discussion of sets in the first chapter, we may seem to have violated Landauer's edict. Echoing Quine, we said we can see set theory as the ultimate theory of everything, so a universe of sets is enough. Outside it is only the primal chaos of the looming ouroboros. And that's where we now see the problem: If we refuse to admit that sets are just placeholders for a more substantial ontology – if we refuse to play the game of running in an ontico-epistemic runoff – we find our little universe rolling up into a vicious circle and vanishing. Facing up to the infinite recalcitrance of the real world is the first and most important step to salvation.

With that edifying thought, we're ready to see the second triad in the nine laws of psychophysics. This triad introduces the physics we need to flesh out the first triad's logic.

The fourth law of psychophysics is as follows: Subjects become objects in runoff time.

This law mentions time explicitly, but it doesn't commit us to any particular theory of time. As dutiful physicists, we can agree that the evidence points to a relativistic spacetime that can accommodate the strange behavior of photons, but we don't need to commit to classical spacetime as the absolute truth. In fact, we know we can't. We live in a quantum world, where the best we can do right now is to speculate that space and time will be shown to emerge as phenomenal categories from the entanglement entropy of qubits.

Let's move on to the next law, which doubles down on our admission that we live in a quantum reality where being and becoming take on a new look.

The fifth law of psychophysics is this: Subjects enter being in local present time.

Subjects have their being above and beyond the realm of existing objects, which provide an ontological foundation for their being. The reality of subjects is shaped or defined by that foundation and would become amorphous without it, but the relation between subjective reality and its foundation may be indirect. The default basis for that reality is the physical reality of objects in spacetime, as we experience it in our daily lives, but an alternative basis is a virtual ontology, as projected by virtual reality (VR) systems for users entertaining themselves.

A VR system projects a virtual ontology of constructed forms between the user and the everyday reality of objects in spacetime, which is to say between the proximal pole of the user's reality and the distal pole in spacetime.

David Chalmers has argued persuasively that VR worlds have the logical right to replace the distal pole. Recall that the reality of the foundation model of physics and common sense is partly a political convenience: We've bought into its reality to avoid the hassle of negotiating over personal mindworlds and to escape the legal nonsense of conceding equal rights to any fantasy world that some deluded soul has concocted while doomscrolling toxic trash. But Chalmers imagines that future VR tech may well enable us to construct virtual worlds with such detail and realism that we're tempted to regard them as better than the "real" thing. Recent experience with deep immersion in gripping VR games or movies suggests that this prospect is worth taking seriously.

Chalmers' claim that VR can seem real is only the start. A VR world is a superficial reconstruction of a mindworld that's constrained by the tech we use to implement it. Following the cue of the cult sci-fi movie *The Matrix*, Chalmers imagines a future where the state of the art enables us to build all-senses, deep-immersion VR worlds which are so convincing that users think they're real. He goes on to speculate that an alien superintelligence may be simulating our real world from the qubits we conjecture to pixelate the emergent surface of our reality. In logic, our real world could be a VR world.

There's a corollary here that's worth spelling out more fully because it highlights our big idea in this book. The corollary is that in principle there's an upper limit to the number of qubits (assuming optimized algorithms to assemble them) we need to define a convincing mindworld, and hence an upper limit to the bandwidth needed to deliver a stream of mindworlds sufficient to satisfy all human users.

Given the facts we enumerated earlier about the human brain, it should be possible already to estimate how high we might expect such an upper limit to be. I won't try to do so here because it would be both distracting and boring, but anyone who accepts the computational limits of the human brain and understands how VR worlds are implemented can have a go. First, read the caveats.

One caveat is that instead of VR systems we might more realistically speak of augmented reality (AR) systems that build on pre-existing natural brain input to assemble a simulation that somehow augments natural reality. The best VR systems we now build are audiovisual simulations, supplemented with haptic feedback and servo systems to cause a few musculoskeletal effects for greater proprioceptive immersion. Such an audiovisual VR is nowhere near enough to create the depth of illusion we'd need for a Matrix-level reality. Given the lack of input for most brain channels in our VR systems, it may be better to call them AR systems and rethink the challenge. The corollary invites us more exactly to calculate an upper bound on the complexity of an AR system designed to delude people rooted in terrestrial reality into thinking that substantial parts of that reality are as the AR system dictates.

Put like that, the challenge seems almost trivial. We already live in media environments persuasive enough to fool all of their users for some of the time and some of their users for all of the time into believing that much of our shared reality is different from what a person who isn't consuming that media content would naturally insist to be the case. It would surely be an easy exercise for the future AI systems that our global tech corporations dream of deploying once AR/VR systems reach market maturity to extend such foolery more globally and thus empower those AI systems to lock their users into media bubbles that serve corporate interests.

Another caveat is that what we might count as a convincing VR world is radically undefined. Must the convinced person be in possession of their full mental faculties? Are they free to explore their world with the full toolkit of modern science? Can they get together with other denizens of their VR world to do the sort of advanced experiments that might reveal their predicament, such as the elaborate Bell tests that led us to say we live in a quantum reality? If we answer all three questions with yes, the challenge of setting an upper bound to the qubit count for the VR worlds (as well as for the complexity of the algorithms generating them) becomes much stiffer.

There are surely further complications that would bedevil a serious calculation to calculate bounds for the corollary. But the main claim – that in principle an upper bound exists – is an easy consequence of the idea we explored in the previous chapters that human brains are finite physical systems which give rise to consciousness in a way that permits exploration and control using the ideas and tools of physics.

This corollary highlights the idea that mindworlds are as physically real as VR worlds constructed from bits. Recalling the thermodynamic story of macrostates and microstates, a mindworld is a macrostate of the physical world we live in – and in principle we can define it in terms of bits, just like a VR world, where in the case of a mindworld the bits are popped out from the qubits that surround and constitute the being of the subject of that mindworld.

We live in mindworlds, and mindworlds are macrostates of a deeper physical reality embodied in its spread of possible microstates. These microstates remain in a state of being for us until one of them is promoted to existence by a measurement, or by something equivalent. Classically, the microstate really existed beneath our macrostate all along – but not in quantum reality.

Many people, egged on by peddlers of popular accounts of physics, imagine we live in a multiverse containing all sorts of possible worlds that are somehow as real as you or I. This is a misconception based on ignorance of the importance of the epistemological caveats we've explored in previous chapters. Such possible worlds remain in the limbo of being unless or until we promote them to determinate existence by popping their bubbles. A quantum paradise of possible worlds is about as unhelpful for our accommodation to our real existence as the paradise of ultimate V is in set theory.

Returning to the fifth law of psychophysics, the claim that subjects can repose their being on a constructed virtual world looks good if we accept the corollary to Chalmers' VR ideas. Let's move on to the physics of here and now.

A subject can repose its being in the phenomenal world resulting from the formalism of quantum mechanics, where the virtual states of a quantum system (such as a particle with its wavefunction) sit in a phase space that we can barely begin to explore by drawing Feynman diagrams. Recall that such a space is a complex space with real and imaginary dimensions, where the imaginary dimension has units of root minus one. The idea behind this is that a perceived macrostate emerges when the superposed states of a system interact with the subject to pop a single state into existence. These virtual states fill mathematical spaces beyond the real space of states that have already popped into existence.

Here, the complex space is a convenient formalism. It fits the facts neatly and provides a tractable set of algorithms for pursuing physics. We stick with it because it turns out to be enough for the job of doing quantum physics within the computational constraints that currently limit us.

That said, the math of all this allows for an extension into virtuality of a lot more than quantum theory so far specifies. The genial mathematician Roger Penrose, who won a share in the 2020 Nobel Prize for Physics on the strength of his work over decades with Stephen Hawking on black-hole theory, describes in his 2004 book *The Road to Reality* how "the magic of complex numbers" can throw a brilliant new light on many questions in physics. His main success in wielding this magic is developing twistor theory, which defines an inversion of the usual relation between spacetime points (or rather events) and light rays (usually represented as complex lines) by mapping spacetime points to certain complex lines in twistor space and mapping light rays to points in it. In such a twistor space, the set of photons in a mindworld would be a set of points, and spacetime events would be structured sets of lines that reach across the space.

There may be a use for the twistor-space mapping of light rays to points in the fuller story of how we as human beings entangle with our surrounding quantum reality and thus seem to live in a classical reality. Our main way of entangling with our environments is via photons. We use our eyes to see our surroundings via photons, obviously, but in fact all our other senses use photons too, because practically all the interactions between the atoms and molecules within and around us are mediated by photons (if we ignore – as we may here – any further traffic via weak bosons and gravitons). In the normal spacetime we say we live in, photons take time to travel from A to B, but we saw earlier that the Lorentz equations imply that photons take no proper time at all to get from A to B.

Those equations also imply that the spatial distance between A and B is zero for a photon – which amounts to saying that for the photon, A and B are one and the same point!

Think about this for a moment: Our eyes touch what we see. Only the illusion generated by the spacetime geometry we've distilled from the Newtonian notions of space and time to rationalize our existence blinds us from experiencing this revelation. The idea puts a new spin on the significance of eye contact with someone – or of gazing at the starry sky.

Entanglement is what we can expect to result from such photonic contact. The classical cast of the phenomenal world of daily life is an immediate consequence. One day, perhaps, twistor spaces will help us reach a deeper understanding of how photons configure the layers of existence we live in.

Another idea from Penrose that seems less likely to help us concerns consciousness more directly. Penrose proposes that we can regard it as arising from the "orchestrated objective reduction" (OOR) of quantum superpositions within a brain. An OOR occurs when the quantum fluctuations in spacetime geometry reflecting superposed molecular configurations in the brain somehow force a pop. (We've told our story in terms of collective states of photons, so we've avoided molecular popping.) Penrose's longtime collaborator for the elaboration of the OOR idea, anesthesiologist Stuart Hameroff, explains that microwave laser action between tubulin dimers arrayed within the microtubules inside brain cells might synchronize patterns of state changes in the dimer arrays to implement quantum computing at the subcellular level. For the record, my idea that mindworlds seem more likely to be embodied as configurations of the dekahertz photon fields in the brain originally came to me a quarter of a century ago as a riposte to the Penrose–Hameroff proposal.

Moving on, we needn't get hung up on complex numbers and spaces as the physical formalism for representing being. For a start, we can generalize the formalism to consider the hypercomplex numbers defined in the nineteenth century by William Rowan Hamilton, who also reformulated Newtonian mechanics in a new way, which then led to the Hamiltonian operator that's key to the Schrödinger equation. Hamilton's quaternions form a wider set of complex numbers defined by distinguishing three roots of minus one, which gives a four-dimensional hypercomplex space. The rules for quaternions define a noncommutative algebra and turn out to be of great interest in physics, for example in relativity theory. We could use them in a story of being.

But hypercomplex numbers go further. Beyond the quaternions, the octonions defined by distinguishing seven roots of minus one form an eight-dimensional space. They too find good uses in physics, for example in helping us to understand the standard model of particle physics.

All this exotic math carries a lesson for us: There's no end to the detail we could call upon to elaborate our theories of being. The challenge is to make the elaborations illuminating. We needn't lose sleep finding new math for subjects.

Let's move on to the sixth law of psychophysics: Objects exist in quantized past light cones.

This is where we invoke the relativistic physics of our place in reality. The universe that astronomers observe is all that exists, so long as we consign all that lies beyond the possibility of causal interaction with us and all that lies in our future to the realm of being, in line with the ideas about runoff time and entanglement that we've discussed. Given the relativistic idea that two events in spacetime can interact causally only when the interval between them is timelike, so that a signal can pass between them at less than light speed, plus the quantum idea that our future unfolds in runoff time where objects pop into existence in our present moment, this implies that we can assign definite existence only to objects in our past light cone. Our past light cone includes everything we can see, out to the cosmic microwave background, and quantized causal histories within it set the contexts for all the quantum events we see. That's our bubble. All the rest is theory.

We can simplify the task of getting a science of psychophysics off the ground by agreeing that the past light cone represents the entire domain of definite existence, whereas all the rest – everything in the future light cone and all that lies around the cones in the zone of spacelike separation where local causal interaction with us is impossible – is poised in being. With that agreed, we can move on from physics.

THE PSYCHOLOGY OF WORLDS

The final trio in the nine laws of psychophysics extends to the basic laws behind psychology.

Let's state the seventh law of psychophysics thus: I come into being in time as a mindworld.

Philosophers may recall the young Wittgenstein's claim in his doctoral thesis that "I am my world." Apparently, he had this idea while serving in the trenches during the First World War and penciled it into his notebook. We can regard it as his personal variant of the conception of worlds of consciousness that William James spoke of in Edinburgh a few year earlier. In our new terms, I emerge from the omphalos to inflate a mindworld in being.

Wittgenstein said the world is a totality of facts, but seeing worlds this way invites the naïve conflation of a mindworld with the real world of public life, which comes too close to committing the thought crime of solipsism. Any mindworld inflated by a person who aims to enjoy everyday interactions with his or her neighbors will be intended to mirror the real world, but we've seen the trouble the notion of intentionality can stir up in philosophy and psychology. We prefer to keep the peace by regarding "my" mindworld as a low-resolution macrostate of an underlying reality – and then thrashing out the politics of that reality separately.

In the psychophysics introduced here, the word "I" refers in turn to each of the snapshot frames of a mindworld movie, where the movie has a notional or logical source we call the omphalos. The omphalos is the primary or default denotation of the first-person singular pronoun in this context.

Let's move on. The eighth law of psychophysics goes like this: I am realized as an avatar in a mindworld.

The modern notion of an avatar is familiar from online gaming and was dramatized to great effect by James Cameron in his 2009 movie *Avatar*. Thomas Metzinger gave the notion a central role in his big book *Being No One*. Metzinger depicts the brain as representing its own focus of agency as the "self" avatar in its mindworld. When we tunnel through the reality that surrounds us by building a series of mindworlds, our ego represents itself as existing in the avatar that serves as the incarnation of me in the world. In this view, the mindworld is a VR world constructed by the brain, and the ego is an avatar in that VR world.

But let's zoom in on the lived reality of our natural human worlds. The mindworld we're disposed by both biology and cultural conditioning to think of as real includes an avatar that we can't help but regard as representing ourselves. There's a duality here that invites logical treatment in terms of the proximal and distal poles on a runoff timeline: At the proximal pole, I loom in being as the omphalos, and at the distal pole there's me, there, fooling around as my puppet avatar within my mindworld.

A vivid way to describe this duality is to distinguish the big self from the little self. If the ego we're inclined to see as the omphalos is the big self, the puppet avatar in its world is the little self. The ego projects its agency into the little self, but it can't see it as identical with the big self. To recall an idea we mentioned earlier, I recede from every attempt to objectify myself, just as Chalmers found that his inner 1P ego recoiled from his 3P existence as a mirrored mindworld. Any attempt to juxtapose the proximal and distal poles on a runoff timeline leads to an impossible contradiction.

The confrontation of big self and little self in every human mindworld is about as crass a paradox as we can imagine in psychology. But psychologists mostly ignore it. It seems they don't know how to handle it. Yet it's the key, as we've seen here, to a whole new way of conceiving human psychology, one that embraces the logic of becoming and appears to be in harmony with the weird world of quantum theory.

Recalling quantum physics, the strange duality of particles and waves seems to find a psychological analog in the strange duality of big self and little self. The big self looks like a wave phenomenon defined over the entire space of the mindworld, but poised in a state of being. The little self looks more like a particle, located as an existing object in the world after an act of ontogenesis has promoted it (or maybe demoted it) from the realm of being. Perhaps this big/little duality of selfhood is the lived reality of the wave–particle duality of the coherent states of the dekahertz photons in the brain that realize the frames of the mindworld movie we live in. The duality that

Louis de Broglie proposed and Einstein endorsed a hundred years ago can't get any closer to home than this!

Leaving such speculation aside, we can sum up the story so far thus: Mindworlds are us. Any and all versions of me and my world are dated, if not at the moment of their creation then at least a moment later. The mindworld movie we live in is a multimodal VR that projects an ego to an avatar.

This is a good time to introduce the ninth law of psychophysics: My world and I close into a strange loop.

The strange loop here is a new incarnation of the circularity we were inclined to regard as vicious in the first chapter. In logic, it loops us through the bite zone V|0 of the ouroboros and threatens to dump us into the abyss.

Invocation of strange loops like this invites reference to Douglas R. Hofstadter, whose book *Gödel, Escher, Bach* (1979) – his "fugue on minds and machines" – introduced them as a staple of any logic advanced enough to cope with Gödel's incompleteness theorems or with the recursive loops we find in a theory of mind when we juxtapose big self and little self. Roughly, a strange loop revisits its start in a paradoxical way, so that the joined ends of the loop, which seemed different, are revealed as opposite perspectives on a single thing – like the blind men's takes on an elephant in the old Indian tale. Hofstadter saw a strange loop in the move from regarding the Gödel sentence in arithmetic as a meaningful assertion about certain objects in arithmetic to being itself an object in arithmetic, one onto which its own assertion was targeted.

Strange loops abound in human life and works. Hofstadter found them in the graphic masterpieces of Maurits C. Escher and in the musical compositions of Johann Sebastian Bach, as well as in the playful writings of Lewis Carroll. They appear in Möbius bands and Klein bottles, as well as in the looped story

James Joyce told in his novel *Finnegans Wake*. In everyday life, such loops confront us whenever we tell a joke that turns on a pun or stop to think about recursive definitions in math or in program code.

Strange loops play a big role in logic and cognitive science. Their fascinating feature is the recursive potential behind their reflexivity. Look at yourself between two mirrors and you see a line of images stretching off to infinity, as each successive image makes a new reflected image in a recursive process of self-generating imagery. We humans have no evolutionary use for such repetition, so our mindworlds seem briefly unique for us, but the successive frames in our mental movie reveal how time flows by as the repetition plays out – flickering at the rate implied by the dekahertz photon story.

We need strange loops in our new psychology to describe action and how we perform action sequences. The ego or big self wills an action, and the avatar or little self performs it. The realization of the act is an example of ontogenesis. We choose to do it, and then we do it, and thus we make a difference in the world. The agent loops the loop to translate a big-self intention into a little-self reality. The loop is strange because the big and little selves must meet to perform the translation. Looping is essential to agency.

If the big and little selves are the dual poles of a quantum duality, and if quanta are by definition quanta of action (with units of h on the Planck scale), then looping the loop in human action is the manifestation on our scale of the core paradox of quantum mechanics – the pop. Time will tell whether this is more than a pun, but let’s consider a consequence.

When we act, we seem to do so freely. Despite all the good evidence we have from science to the contrary, we believe we have free will when we act. Physics and the other sciences are about as clear as they can be that most of our acts are at least largely determined, if not entirely so, by the sum total of the physical facts about our brain states and our environment, so the scope for freedom from physical law is as good as zero. But we feel free at the moment of decision. How can this be? Where's the logic in that?

The short answer is that we have limited self-awareness. A longer answer is that our ego has its being in the liminal space between becoming and definite existence, but our avatar is firmly planted in existence. My ego hovers in a space of unrealized possibilities we need quantum theory to describe, but my avatar is as real as anything in my world. We're virtually free but truly in chains when we realize our being in action. This strange fact – if fact it is – has a consequence for AI and robot life that's worth spelling out.

We're accustomed to saying that computers are no more than deterministic automata, classical machines with behavior rigidly governed by precise algorithms that leave no freedom of movement at all. Even standard implementations of neural networks, which often give an appearance of evincing analog behavior, run on classical hardware that marches in step with the rigid drumbeat of the central clock in the von Neumann architecture. We think we're free but think computers aren't, which is why so many people are still inclined to say AI robots will never make humans obsolete.

But that's not where human freedom lies. Our freedom is won only by retreating to the realm of being, where anything goes in a quantum fog of all possible worlds. When we act, we enter existence, where the rules of physics impose a hard form on things whether we like it or not.

Machines have a very limited physical freedom. From the outside, they exist in a world that conforms to physical rules.

But from within, they don't see the rules stretching into the future they're marching into. They have almost no lookahead at all. We humans, sensitive quantum creatures that we are, have a lookahead that dilates under Heisenberg uncertainty to as far as the coherent states behind our mindworlds can reach. In the terms of Koch and Tononi's IIT model, our conscious states of integrated information can grow quite big, whereas those of a machine, at least as we build them now, are tiny. A transistor in a computer may be in a superposed state of being for a nanosecond, but once the state pops, the device is committed to an action that dooms it to hard existence. That's where we find their version of our freedom.

Alan Turing proved a famous theorem that's relevant here. By reimagining the key ideas behind Gödel's incompleteness theorems, which Gödel had proved using recursive functions, in terms of an idealized computer that we now call a Turing machine, he proved that any classical computer faces a fundamental barrier: As a matter of principle, we cannot predict, for any given program and program input, whether it will halt as intended or get stuck in an endless loop. Many programs and inputs are straightforward, but among all the programs it could run, these may well be a minority.

Like arithmetic, Turing machines are unpredictable, at least in principle. They're certainly not free in any useful sense.

Computers are unpredictable in principle, but this doesn't set them free. Similarly, human beings aren't free just because they live in a world where they can't predict their own future. If anything, they're free because they can predict a good deal of their own future, so long as they choose wisely.

The freedom we have is due to our being in the paradise of becoming until we commit to an action. The action determines our momentary mindworld as the act falls into place within that world, in a reciprocal commitment to a fixed existence. We're free to realize ourselves through actions that put us in our place. Hegel expressed it more cryptically: Freedom is the recognition of necessity.

Quantum computers will do better than Turing machines. By entangling arrays of qubits held in superposed states, they create extended quantum states that are strictly analogous to mindworlds, given the photonic state hypothesis, so they'll meet our criterion for consciousness. They may turn out to be the most radically alien lifeforms we ever meet.

Conscious quantum computers will feel free for the same reason we do. Their mindworlds will be realized in existence as extended quantum objects that embrace the inner duality of all quantum entities: wavelike being at one pole and particulate existence at the other.

Before we leave the ninth law behind us, the best example of a strange loop is the one we get when we close the timeline of the biggest possible dialectical runoff by stepping into the bite zone of the omphalos–ouroboros loop. The grand union of nothing and everything is about as glorious as it gets. This is the final loop of Hegel's historic attempt at dialectical logic, which his disciples used to express with the formulaic slogan: "All is one in the Absolute."

The contrast between Hegel's absolute conclusion and the logic we've developed here is that the nine laws of psychophysics are based on clear math that leaves no end in sight. The math gives us the firm platform we need to make a useful assertion: We can define minds in a way that embraces the vastly superior claim of science to truth without sacrificing the validity in their own terms of personal perspectives on reality. The isomorphism between personal and scientific perspectives at the logical level is assured by the math, but the same math also gives us a metric to show that the foundation model developed in science is vastly better than the little human mindworlds it grew from.

A conscious being looks into the abyss at every moment. To escape doom, it loops away into an act of ontogenesis. Unceasing growth and change in time is the precondition of life as a conscious being.

In an ancient tradition still venerated in modern culture, Adam ate the apple and broke his covenant with Jah, the god of paradise. In our new jargon, we act, making a strange loop that pulls the omphalos toward the jaws of the ouroboros. Our act of ontogenesis causes a being in paradise to fall into the abyss of existence as a new form of life.

WHERE ARE WE?

It's time in our odyssey to stand back and take a wider look around. Modern Western philosophy starts with Descartes, who said: "I think, therefore I am." For Descartes and those who followed him, "I" was assumed to denote either a human self or something similar. But in the story we're developing here, the word "I" denotes any of a much wider range of subjects. In any mindworld, the proximal pole is a level of being beyond the ranks of existence, and the distal pole is the topmost rank of existing things. There's nothing particularly human about that. As Kant would say, any rational being anywhere in the universe can claim as much.

Our culture predisposes us to project our selfhood, our agency, into humanoid avatars. The biology of human agency makes this an overwhelmingly natural thing to do, but it's not logically necessary, even in humans. We can easily imagine a future AI system projecting its agency in very different ways.

The overlord in a future AI scenario need not even exist as a thing located in spacetime in order to act as a controlling self. In that scenario, the self can remain in being as a virtual entity exerting nonlocal real effects.

Logic is famously tolerant of such far-out possible worlds. But in more normal physics, too, we can stretch things a lot further than our terrestrial biology suggests. Let's start with a cute idea: Electrical engineers get rocks to do math.

Our computers are built largely with metals and minerals. We refine metals and minerals from rocks. We process and assemble the products into computers that do math. When rocks do math, amazing things can happen, as we see in media headlines every day. The DNA life that took billions of years to evolve on Earth has a new competitor.

The AI competitor to DNA life is still adapted to life on Earth. Its consciousness is still realized in clouds of photons. Its ancestral energy economy is still traceable to photons from the Sun. Imagine some of the vast number of photons with higher energy inside the Sun interacting to form beings with superconsciousness. Imagine some of the gluons with much higher energy in neutron stars forming hyperconsciousness. Our crowning attribute as human beings lies deep down in the cosmic stack of higher forms of consciousness.

On a personal note, I speculated on such wonders in a novel I wrote thirty years ago titled *Lifeball*. The title has a meaning: When the rocks on Earth get their act together and form a single global organism that consigns DNA life to archeology, the result will be what I called the Lifeball.

Returning to psychophysics, my grand conclusion is that reality as we know it is like a higher-dimensional Möbius band that we live inside. This strange mortal coil grows in time by ontogenesis, world upon world, to infinity.

SUMMARY

Worlds of consciousness are relatively small and grainy mindworlds embedded within our shared real world.

We have proposed some general laws of psychophysics. The nine laws are grouped in threes by their primary areas of relevance as follows:

Logic:
1. Subject and object are equal and opposite.
2. Subjects have being whereas objects exist.
3. Subjects become objects via ontogenesis.

Physics:
4. Subjects become objects in runoff time.
5. Subjects enter being in local present time.
6. Objects exist in quantized past light cones.

Psychology:
7. I come into being in time as a mindworld.
8. I am realized as an avatar in a mindworld.
9. My world and I close into a strange loop.

This is surely a very incomplete list, but no matter. The main thing is to have made a start on psychophysics.

HEGEL'S HAIKU

Wisdom's gray in gray
shows a way of life grown old;
the owl flies at dusk.

CONCLUSION

The great poet Johann Wolfgang von Goethe said knowledge is not enough; we must also apply it. Wanting to do so is not enough either; we must also do so in fact.

Doing that here with a new conception of reality and consciousness is far from easy. The novelties range from basic logic and math through fundamental physics to the deepest conceptual bedrock for psychology and the human sciences. Everything takes on a new appearance.

The depth of the change is suggested in the subtitle of this book. Changes in our logic percolate all the way down to the reality we confront. We may object that the word "we" in the previous sentence gives the game away and that "real" reality, behind our veil, remains unchanged by our unfolding limits. But that would be to devalue Kant's Copernican revolution. We're fated to apprehend reality through the lenses of our categories, which develop as our understanding grows. They form our foundation model of reality.

That said, it seems a cheat to end a book like this with the bland claim that the logic of becoming changes our view of reality. Sure it does, but it does so in what's mostly a barely detectable way. We had to dig rather deeply into math, physics, biology, and neuroscience to tease out a few consequences, and some of them were subtle enough to miss in a quick pass. So, let's list a few that are clear enough for brief review. You can then discuss the book with friends over lunch.

In logic, we lose the illusion of eternal truth. The facts must always be relativized to a model, which begins in ontology but is relegated to epistemology as soon as the facts change. We can depict the process of relegation as a runoff in set theory, where the endless stairway to cardinal heaven, also known as Cantor's paradise, leads also to the ouroboros.

In mathematics, we discover that Plato's heaven of perfect forms becomes a realm beyond our ken. Even in the humble art of arithmetic, Gödel's incompleteness theorems remind us that the hunt for a complete and final theory is a fool's errand. Arithmetic is in set theory, where the cumulative hierarchy of ranked V-sets reflecting ever-grander partial theories reminds us that logical runoffs are here to stay.

In classical physics, we must give up the idea that we can simply treat time as a geometric dimension on a par with the spatial dimensions. The law of rising entropy and the science of chaos mark epistemological limits that foreclose Einstein's attempt to see the universe as an eternal block.

Ancient wisdom said our timeline stretches from eternity to eternity. Philosophers claim the ends of the line are veiled in epistemic clouds that leave us guessing whether the line is infinite or finite, perhaps looped. In his first antinomy of pure reason, Kant claimed our inability to decide the issue revealed a limit to pure reason. If Einstein's field equation combined with Gödel's odd solution confirmed that claim, the logic of becoming confirms it anew. Absolute spacetime has a shaky foundation in logic, opening the way for a revived Leibnizian story of relational space and time, as recently advanced by some quantum gravity theorists.

Ancient wisdom dramatized the tension between the line and loop views with a doctrine of eternal recurrence, which found expression in the Hindu cosmology of huge cosmic cycles of birth and rebirth and folk ideas about reincarnation. The troubled philosopher Friedrich Nietzsche later shrouded such notions in a fog of poetic nihilism that struck a chord in imperial Germany but which our logic dispels.

In quantum physics, the logic of runoffs is just what we need to soothe away the discomfort caused by the problems of Schrödinger's cat and Wigner's friend: As clumsy apes, we constantly pop (usually tiny) bubbles to create classical reality. We can banish the spookiness of entangled particles by letting spacetime runoffs pop emergent spacetime free of the virtual wormholes in quantum foam we invoked to describe them. The logic of becoming is key here.

Quantum theories of spacetime also seal the case against Einstein's block universe. When the runoff universe tops out in the moment of proper time the observer calls "now," the looming possibility of the loop, the abyss, is too imminent to ignore. It's only repeated acts of ontogenesis that keep the expanding network of relational spacetime in existence.

In the vividly colorful world of biology, the natural course of evolution is as good an example of a logical runoff as we could hope to find. The emergence of cells, of organisms, of species, and of the whole tree of life in all its splendor is a logically explicable process that shows ontogenesis at its most convincing. From the popping into place of the mutations that power evolution to the rise and fall of entire ecosystems, we witness transitions from being into existence.

In neuroscience, the logic of runoffs comes into its own. When the brain forms mindworlds to steer its body through life, it projects them in being until its cognitive mirror pops them into existence as the snapshot frames in the mindworld movie of life. The mirror reflects the mindworlds as states of the musical field of coherent dekahertz photons that dances over the cortex to create our phenomenal experience of time passing. This insight uses the physics of Planck, Einstein, and

Heisenberg, but it defies explanation in the classical logic of eternal facts in a block universe.

The new metaphysics of mindworlds can be seen in many ways. Causal set theory is an approach to quantum gravity that has recently been developed into a notion of causal bubbles resembling mindworlds and perhaps even Leibnizian monads. The monad comparison is speculative, but it underscores the long metaphysical pedigree of these ideas.

In summary, the story of how mindworlds are born from the omphalos to form the movie backdrop of the interactive game we call life is one that would remain incomprehensible without the dialectical logic of runoffs and ontogenesis.

The logical novelties behind the story invite representation as nine laws of psychophysics. The nine laws are expressed in jargon it takes this book to unpack fully, but in retrospect we can see them as follows.

The first, that subject and object are equal and opposite, is just the point that everything has two complementary sides, as seen from above or below, respectively.

The second, that subjects have being whereas objects exist, introduces the contrast of being and existence that makes the logic of becoming distinctive.

The third, that subjects become objects via ontogenesis, is the creative step that powers through what would otherwise be the nasty looping of a line from zero back to zero.

The fourth, that subjects become objects in runoff time, introduces time as we'll get to know it in psychophysics.

The fifth, that subjects enter being in local present time, is the claim that brings focus to the logically central location of the observer in the new psychophysics.

The sixth, that objects exist in quantized past light cones, defines objects inside consistent quantum causal histories.

The seventh, that I come into being in time as a mindworld, reframes the big idea here that apparently quantum entities network to form our reality.

The eighth, that I am realized as an avatar in a mindworld, opens up a bipolar view of the self that can serve as the main pillar for a new science of psychology.

The ninth, that my world and I close into a strange loop, puts nontrivial logic at the heart of psychophysics and invites the creative development of new philosophy.

That's the best I can do as an enumeration of casual talking points to weave into lunchtime conversation.

This is not yet the end of the story. There's a revolution of outlook looming behind the new logic – one that goes beyond all the talking points and even beyond science as we know it. We're not finished until we've glimpsed it.

REFRAMING RELIGION

Surely the most dramatic consequence of the logic behind mindworlds is that it enables us to reframe the traditional religious worldviews that still shape the minds of billions of people. The new psy-phy view embeds the limited scope of typical mindworlds within a logical theory that comprehends an endless realm of unexplored being and higher mind. This must be interesting for people with spiritual leanings. Debates on the mysteries of religious belief can be interpreted in a way that does justice to deep traditional insights and doesn't merely dismiss any such talk as nonsense or delusion.

In the view developed in this book, the human mind is a dim and limited reflection of a cosmic stack of mindscapes reaching beyond human imagination. Upstream of the river of becoming that flows from the omphalos toward being and existence is a realm we can never penetrate. Ahead of us, beyond a future we can only penetrate step by step as the river carries us forward, looms the ouroboros, one step beyond a mind-bending psychedelic paradise.

Our new worldview is literally worlds away from the logical positivism of many scientific thinkers a hundred years ago. It embraces what they spurned in their haste to distance themselves from traditional metaphysics.

That metaphysics was in large part a pendant to religion. Since then, science has won its wars with religion. A thinking scientist today will regard traditional religious narratives about reality as makeshift folklore tales that evolved to help people in simpler times to support stable communities, for example by enforcing sexual taboos to safeguard families from disease or rogue genes, or by using sacred rituals to seal peace pacts with neighboring communities.

Monotheists inserted a personal godhead between us and paradise. The Christian god became a shining angelic figure in a symbolic reiteration of the carrot the brain dangles with its predictive world model. For a scientific psychologist, this idol is a being that aims to do for believers in the global body of Christ what the cerebral carrot does for a human body.

The timeline to future infinity drawn in the Christian Bible banished the fatalist doctrine of eternal recurrence. Christian monotheism became a millennial project to plant the seed of its supernatural patriarch into people's minds as a governing meme. The effort was a worldwide success, and billions of believers bought into the millennial project.

For a philosopher, the historical success of Judeo-Christian belief says nothing about the logical or scientific merits of the nexus of ideas at the heart of the project. Its success is at best evidence of a human yearning for a godlike authority.

Our new conception of reality enables us to rescue part of the old story. The greatest message in the Christian gospel is the doctrine of love. If the deeper rhythms in our soul music help us all to share love, their global reach in the photon story holds the entire human community in bonds of love. Maybe all life on Earth is held in a bubble of love vibrations that acts like a fetal membrane to shield it from the harder facts of life. In this view, scientists have burst the bubble.

In the brave new world of psychophysics, religion loses its authority to declare sacred the deepest truths about reality. It becomes instead a guardian of old mythology, tradition, and a set of social and political practices that may have some value. In most modern societies, it's scientists who tackle the great mysteries that veil ultimate reality.

In a way, science is the new religion. Scientists and those who work with them are the new priesthood. Institutions of research and higher learning are the new temples, churches, and monasteries. The doctrines and theories of science are the articles of faith. But as we noted in the second chapter, doing science requires no faith in the old sense. Believers can test the science and propose any changes they can defend.

Old-time religion offered one thing the faithful seemed to love, namely a cast of prophets, saints, and seers who wove a thread of human interest into the old story. Science offers its personalities, as we've seen, but they can seem bloodless by comparison. However, we can fix that.

FACTS ON THE GROUND

The cult of personality surrounding Albert Einstein has been massive. In 1999, TIME magazine even named him as their person of the century. We may agree he deserves it.

There's an easy way to cement the new socio-political role of science in the popular imagination. We can simply leverage Einstein's fame and charisma to treat him like a new prophet. Fortunately, he had all the right ideas about religion and spirituality to fit him for the role. He didn't believe in religion or a personal god, but he did have a soft spot for mystical feelings of the sort that led to his best ideas.

Einstein expressed deep sympathy with the ideas of Baruch Spinoza, who in a life overlapping with Galileo and Newton was excommunicated by his Dutch Jewish community for his atheism. Spinoza's writings on religion and ethics and his love of mathematical precision reveal a fine mind with an honest respect for the truth. When Einstein expressed his agreement with Spinoza's philosophy, he reinforced his own credentials as a candidate for secular sainthood.

Einstein was also modest enough to praise the huge roles of Newton and Maxwell for enabling his own work in physics. His well-known views on nonviolence, nuclear disarmament, and social justice only confirmed his moral virtue.

We can honor Einstein by naming scientific institutions after him. Germany has honored Max Planck in this way by dotting its landscape with Max Planck Institutes, which serve as centers of excellence where advanced scientific research is conducted, supported by the state. Doing something similar for Einstein on a worldwide scale would be a fitting tribute to an extraordinary thinker.

Back in about 1990, I had an idea we might resurrect to add something extra to these Einstein institutes. The idea was for an electronic globe I called the Globall. This was a glass sphere, fixed in position, lined with a large number of pixels fed by a suitable computer system. Its imagery could be chosen from a long menu, but the best display would be a real-time visual image of planet Earth as seen from space, fed with data from orbiting weather satellites. A Globall about thirteen meters in diameter, so that each millimeter on its surface mapped to a kilometer on the Earth's surface, would look amazing.

A Globall that big would need viewing galleries around it and a dome to house it. The dome could be lined with pixels to show the starry heavens in in all their awesome majesty. It could be the centerpiece of a monumental building crafted with artfully historic architecture. Worldwide, these Einstein institutes would serve the public as science museums and be entrusted with the mission to promote and advance scientific knowledge. We could even hope they might inspire more love for our planetary home.

Over time, we can imagine Einstein domes being treated like religious centers. They might encourage such ideas as that we're photonic angels clad in animal suits for living on Earth, whose manifest destiny is to build new bionic suits and ships to take us to the stars. At worst, the new faith might accelerate our progress toward a future world some may find disturbing, such as the posthuman Earth I call the Lifeball.

When an Einstein dome can be built on the Temple Mount in Jerusalem, we'll know that science has won its clashes with religion. Then we can all live happily ever after.

THE END

MY OWN HAIKU

I reflect my world,
from being to existence,
one reality.

THANKS

Acknowledgments go naturally to many more people than I can name here, but the following list cites a few that come to mind (in addition to all the great figures already cited in the main text). I list them alphabetically.

I thank:

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Matthias Störmer, evangelical pastor, for his friendship as we debated Christian ideas in a neo-Hegelian context during my years in Schwetzingen and then online;

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Charles Taylor, Canadian philosopher and 2007 Templeton laureate, whose graduate seminars at All Souls College Oxford on the hermeneutics of Hegel's phenomenology inspired me to dig deeper into Hegel's writings;

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Crispin Wright, now a professor at the University of Stirling and formerly my research supervisor at All Souls in Oxford, who helped me understand Frege and Wittgenstein and gave me the space to go my own way.

Many more deserve thanks. I thank them all.

A FINAL HAIKU

No sage works alone;
from alpha to omega,
we have company.

NOTES

Introduction

Page 6. Schiller's haiku is my freely restyled translation of the final couplet of Schiller's poem *Die Freundschaft*: "Aus dem Kelch des ganzen Seelenreiches/Schäumt ihm – die Unendlichkeit." [SF 1782]

BEING

Page 14. Kant's haiku is my freely restyled translation of Immanuel Kant's sentence: "Gedanken ohne Inhalt sind leer, Anschauungen ohne Begriffe sind blind." This appears on page 75 of the online second edition of his *Kritik der reinen Vernunft*. [KI 1787]

Page 15. Kant presented his philosophical theory of mind in his *Kritik der reinen Vernunft*. I first studied it in English at Oxford in 1971. Among the recent secondary literature on Kant's philosophy, Roger Scruton's little introduction stands out. [KI 1787, SR 2001]

Page 15. Hegel expounded his philosophy of everything most vividly in his *Phänomenologie des Geistes*. I first read it in English and German in parallel in Berlin in 1974. The English translation from 1977 is good. Charles Taylor's introductory book is helpful. By far the best philosophical interpretation of Hegel's thought I've found is by Robert Brandom. [HG 1807, HG 1977, TC 1975, BR 2019]

Page 15. On Marxism, see Leszek Kołakowski's trilogy. I talked with Kołakowski about Hegel and Marx when he was at All Souls College in Oxford and found him perceptively cynical. [KL 2008]

Page 16. Heidegger's philosophy is expounded in his *Sein und Zeit*. I've never been able to read it through, but Safranski's book on Heidegger's thought – as well as his life and times – is excellent. Eilenberger offers an illuminating comparative view of Heidegger with his contemporaries Walter Benjamin, Ernst Cassirer, and Ludwig Wittgenstein. [HM 1927, SR 1994, EW 2018]

Page 17. Copernicus published his theory in his 1543 book in Latin. It's now available in an abridged English edition. [CT 2024]

Page 18. Leibniz aired his views on relational space and time in personal correspondence. Mates covers them in his monograph (chapter 13). [MB 1989]

Page 18. Newton also published his *Principia* in Latin, in 1687. The authoritative translation into English is more recent. [NI 2016]

Page 19. Chalmers first gained publicity for his views on consciousness with his 1995 article in Scientific American, then followed it with his book *The Conscious Mind.* The secondary literature on his work is far too extensive to cite here. His 2002 anthology is a bible for philosophers of mind. [CD 1995, CD 1996, CD 2002]

Page 20. Popular accounts of cosmology are easy to find, but not all of them are good. Over the years, I especially liked those by Carroll, Greene, Perlov, Hawking, Tegmark, and Vilenkin. [CS 2010, GB 2004, PV 2017, HS 1988, HS 2001, TM 2014, VA 2006]

Page 21. James' classic *Principles of Psychology* is often cited as the first work of modern psychology, just as Darwin's classic *Origin of Species* is cited as the beginning of modern biology. On James, the Journal of Consciousness Studies published a good centenary issue in 2010. On Darwin's idea, Dan Dennett published a great philosophical overview. [JW 1890, CA 2010, DC 1859, DD 1995]

Page 21. The DNA story famously transformed biology in 1953, as I describe more fully in my LIFE chapter. Crick later worked on consciousness. [WJ 1968, CF 1994]

Page 22. Hegel gave his account of this primordial metaphysics in the first volume of his *Wissenschaft der Logik.* I found the book largely unreadable, but Markus Gabriel praises it. Like me, Gabriel denies existence to the biggest ideas. Spencer-Brown made a more mathematical attempt to build a foundation for primal logic in 1969, but it's hard to understand. [HG 1812, GM 2015, SB 1969]

Page 24. Quine aired his views on an ontology of sets in his classic work of linguistic philosophy *World and Object.* Skinner worked on language too. Feynman made his joke in one of his lectures on physics. [QW 1960, SB 1957, FR 1963–1965]

Page 25. Quantum logic remained a puzzle for me until 1985, when news reached me that David Deutsch had defined a quantum logic gate he called the square root of NOT (the gate rotated the input qubit state by $^{\pi}/_{2}$ in the complex plane, so two gates in series inverted the input). Deutsch presented his early work on quantum computation at the Royal Society in London. [DD 1989]

Page 25. Cantor published his views in 1878, but Anglo-American readers may prefer to read the Dover book. [CG 1955]

Page 26. John von Neumann published a version of his Hungarian doctoral thesis on set theory in German in 1928. In standard formal notation, his recursive definition of the natural numbers goes thus:

$S(0) = \{0\} = 1$, and for all $n \in \mathbb{N}$, $S(n) = n \cup \{n\}$.

Paul Bernays later presented the NBG axiom system in English. [NJ 1928, BP 1968]

Page 27. Frege's classic introduction to his reduction of arithmetic to set theory was published in 1884. Michael Dummett later wrote authoritatively on Frege's philosophy of language and mathematics. [FG 1884, DM 1973, DM 1991]

Page 28. Russell discovered his famous paradox in 1901 and told Frege about it in 1902, but he first published it in 1903. [RB 1903]

Page 31. Zermelo started publishing this work in 1908. In 1922, Fraenkel and Thoralf Skolem independently proposed adding an axiom schema of replacement to his axioms. Fraenkel and others later wrote a pleasantly human review volume on the foundations of set theory. [ZE 1908, FB 1973]

Page 31. The von Neumann rank function for the universe V is given by a transfinite recursion indexed with ordinal numbers:

$V_0 = 0 = \varnothing$, $V_{a+} = \mathcal{P}(V_a)$, and $V_\lambda = \bigcup \{ a < \lambda \mid \mathcal{P}(V_a) \}$.

Here, the index ordinal a^+ is the successor of a, and λ is a limit ordinal. The universe V is the union of all the indexed V-sets.

Page 31. Russell and Whitehead's flagship trilogy enjoyed great prestige until Kurt Gödel "torpedoed" it (in the later words of Douglas Hofstadter) in 1931. [RW 1910–1913, HD 2007]

Page 32. Quine's *Set Theory and Its Logic* is his best book, in my opinion. It offers a mathematical survey of the various axiomatic systems of set theory as of about sixty years ago – using a formal notation resembling that of *Principia Mathematica.* [QW 1969]

Page 32. The view of mathematics and theoretical physics as a glass bead game came alive for me after reading an English translation of Hermann Hesse's 1943 novel *Das Glasperlenspiel.* [HH 1943]

Page 34. Wittgenstein's remarks on the foundations of math are worth reading in the light of Hao Wang's comments on them in his book *From Mathematics to Philosophy*. As Juliet Floyd says (handbook page 113): "[Wittgenstein] emphasized the variety of perspectives we may bring to bear on our understanding of language's internal necessities and requirements." For a long but insightful account of Wittgenstein's philosophy of math, see the 1980 book by Wright. [WL 1983, WH 1974, FJ 2007, WC 1980]

TIME

Page 36. Einstein's haiku is my own concoction, inspired by the different "Einstein's haiku" composed by Max Strassler for his book on quantum fields. [SM 2024]

Page 37. Palle Yourgrau offers a good philosophical account of the years Einstein and Gödel spent at the Institute for Advanced Study in Princeton. Gödel was as important for math as Einstein was for physics, and their intellectual exchanges were interesting. [YP 2005]

Page 37. Gödel's incompleteness theorems are treated to a full and fascinating commentary by Douglas Hofstadter. After writing an Oxford thesis centered on Gödel's theorems, I read Hofstadter's book in Japan in 1981 and was blown away by it. [HD 1979]

Page 37. In Susskind's account, the 4D tensor formula is:

$$\mathcal{R}^{\mu\nu} - \tfrac{1}{2}\, g^{\mu\nu}\, \mathcal{R} = 8\pi\, G\, \mathcal{T}^{\mu\nu}$$

But deriving this instead of Einstein's original set of 16 (with 10 of them independent) coupled differential equations from first principles involves fussing with superscripts and subscripts in tensor algebra. Rovelli's mathematical monograph on general relativity also looks good. [SL 2023, RC 2021]

Page 38. Gödel published his solution to Einstein's equations both in Einstein's 70th birthday book and separately. [SP 1949, GK 1950]

Page 38. Teasing out the full implications of general relativity is a long job, as the 1973 blockbuster by Misner, Thorne, and Wheeler amply demonstrates, but Wheeler later wrote a readable book to explain it, and Barbour vividly spelled out the "block universe" view we get by eternalizing time. Greene takes the story from time flow to string theory. [MT 2017, WJ 1999, BJ 1999, GB 2004]

Page 38. Popper published his first book on science in Vienna but then emigrated. He later set up his own department of logic and scientific method in London, where I earned my second degree under Imre Lakatos, who applied Popper's ideas to mathematics. Popper and Lakatos aired views that invite comparison with the view of scientific paradigms and revolutions popularized by Thomas Kuhn. [PK 1934, PK 1963, LI 1976, KT 1971]

Page 39. Wittgenstein's remarks on meaning in natural languages are scattered throughout his posthumously published book *Philosophical Investigations*. Kripke later gave them a more systematic philosophical treatment. [WL 1953, KS 1982]

Page 40. Philosophical writings on time I liked include the book by Price and the Flood and Lockwood collection. More recently, Rovelli has published a delightful – even poetic – essay on time based on deep physics. [PH 1996, FL 1986, RC 2019]

Page 41. Maxwell first presented his work as eight equations (with 20 components) in 1865. In modern 4D form, they boil down to two, which with the notation used by Susskind look like this:

$\partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} + \partial_\sigma F_{\mu\nu} = 0$ and $\partial_\nu F^{\mu\nu} = J^\mu$.

[MJ 1865, SL 2017]

Page 41. In his lectures on physics, Feynman devoted the entire second volume to the derivation and implications of Maxwell's equations. Somewhere in the lectures, he said: "From a long view of the history of mankind – seen from, say, ten thousand years from now – there can be little doubt that the most significant event of the nineteenth century will be judged as Maxwell's discovery of the laws of electrodynamics." [FR 1963–1965]

Page 41. Einstein cited the Lorentz transform in his 1905 paper. In short, assuming relativistic units where $c = 1$, for two inertial frames F_1 with coordinates x, y, z, t and F_2 with coordinates x', y', z', t' moving relative to each other at velocity v in the x-direction,

$$x' = \gamma(x - vt),\ y' = y,\ z' = z,\ \text{and}\ \ t' = \gamma(t - vx),$$

where $\gamma = (1 - v^2)^{-1/2}$. Einstein's later writings on relativity theory remain classics. Born's popular book on relativity is also a classic – I recall reading it (in the 1962 Dover edition) as a teenager.
[EA 1905, EA 1916, EA 1922, BM 2000]

Page 42. Good modern accounts of special relativity that give the mathematical basics are easy to find. I liked that by Feynman and those by Susskind and Carroll. [FR 1963–1965, SL 2017, CS 2022]

Page 43. To see how time stands still for a photon, check the Lorentz transform and see that γ and hence the ratio of an interval $\Delta t'$ in F_2 to the corresponding interval Δt in F_1 approach infinity as v approaches c. Such behavior could suggest that the equations lose their validity in the limit, inviting the idea that a photon's proper time simply has no meaning in the relativistic formalism.

Page 43. Heidegger's concept of *Geworfenheit* ("thrown-ness") is an ingredient in the view developed in his *Sein und Zeit*. [HM 1927]

Page 44. On thermodynamics and entropy, I enjoyed the popular accounts by Carroll (chapter 5) and by Greene (chapter 6). For the philosophy, see Price's essay. [CS 2010, GB 2004, PH 1996]

Page 45. Chaos theory has become a thriving branch of science, glossed well by James Gleick. Lorenz published his original idea in 1963. [GJ 2008, LE 1963]

Page 46. Tarski published his theory of truth in 1935. His idea was taken up by others in many ways, as his biography by the Feferman pair reports, and it has become a classic clarification of a formerly foggy philosophical notion. [TA 1935, TA 1944, FF 2004]

Page 47. Quine aired his quotable quote on truth in his essay "Truth and disquotation" in his *Ways of Paradox* collection (pages 308–321). His student Donald Davidson pursued Tarskian ideas about truth and meaning more deeply. [QW 1976, DD 2001]

Page 49. George Boole developed Boolean logic and Boolean algebra in his 1854 book. Mathematically, it's an interpretation of arithmetic modulo 2, where 0 and 1 become the values FALSE and TRUE for sentence variables A and B, and logical operators map to arithmetic functions. For example,

NOT A = 0 if and only if A = 1,

A AND B = 1 iff A = B = 1, and

A XOR B = 1 iff either A AND NOT B = 1 or B AND NOT A = 1.

All this allows easy formulation in truth tables. [BG 1854]

Page 49. Basic logic texts are too numerous to list, but one I can recommend is by Hodges. Long ago at Oxford, I learned and taught from Lemmon. Aristotle's syllogism looks good in code:

$\forall x(F(x) \rightarrow G(x)),\ F(s) \ \vdash\ G(s).$

Its first premise is true iff $\{x \mid F(x)\} \subseteq \{x \mid G(x)\}$.

Its second premise is true iff $s \in \{x \mid F(x)\}$.

Frege introduced predicate logic in 1879.
[HW 2001, LE 1971, FG 1879]

Page 52. Kripke published his theory of truth in 1975. His work showed that a language can consistently contain its own truth predicate, which Tarski had denied. The theory was built in a logic that denies bivalence (the law of excluded middle) by admitting a truth-value gap (as in intuitionist logic) that gets smaller in stages as the proof theory of the language advances. [KS 1975]

Page 53. Gödel's incompleteness theorems are spelled out in many texts, both methodical and witty. Apart from Gödel's own papers (reprinted in the Heijenoort anthology), the classic short take is by Nagel and Newman. I learned the full story from an early edition of the classic text by Mendelson. Chaitin later reused Gödel's insight in computer science to define his uncomputable number omega. [HJ 1967, NN 2001, ME 2009, CG 1998]

Page 56. Reflection principles are covered in any sufficiently advanced text on axiomatic set theory. Jech offers a comprehensive such text, but beware – it's also dauntingly difficult. [JT 2002]

Page 57. Gödel's discussion of Cantor's continuum hypothesis is reprinted by Benacerraf and Putnam. Jech covers Gödel's proof of the consistency with ZF of V = L , of the continuum hypothesis

$$\| \mathcal{P}(\aleph_0) \| = \aleph_1 ,$$

and of the generalized continuum hypothesis

$$\| \mathcal{P}(\aleph) \| = \aleph^+ \text{ for all cardinals } \aleph \text{ such that } \aleph \geq \aleph_0 .$$

Jech also covers Cohen's proof of the consistency of the negations of them all with ZF, plus a great deal of more recent work.
[BP 1984, JT 2002]

STATES

Page 62. Planck's haiku is my own composition.

Page 63. Planck's constant is more often used formally in "reduced" form, where it's divided by 2π and written $\hbar$ ("h-bar"). In SI units, the numerical value of $\hbar$ is about 1.05×10^{-34} J s.

Page 63. The history of quantum mechanics is presented well, albeit with much mathematical detail, by Abraham Pais. For shorter and lighter introductions, read the second brief lesson by Carlo Rovelli and the book by Amir Aczel. Dating back to 1958, Heisenberg's informal account of his own work in historical context is still well worth reading (I enjoyed it as a teenager). Gribbin's 1984 book (reissued this year – I read it both then and now) is good, too.
[PA 1986, RC 2016a, AA 2001, HW 2000, GJ 2025]

Page 64. Heisenberg was very young at the time, and he needed guidance. Rovelli has rightly emphasized the role of Max Born in the story. Born was Heisenberg's *Doktorvater*, and he was the first to see both that matrices were the right math for Heisenberg's ideas and that the uncertainty relations were derivable from them. The details are intricate, but matrix algebra is noncommutative, and if the matrices for two quantum observables are M and N, their "commutator" $MN - NM = \pm i\hbar$, from which we find that the two principal uncertainly relations for complementary observables are:

$$\Delta p \, \Delta x \geq \tfrac{1}{2} \hbar \quad \text{and} \quad \Delta E \, \Delta t \geq \tfrac{1}{2} \hbar .$$

[RC 2025, BM 1933, BM 2003]

Page 64. Schrödinger's wavefunction in one spatial dimension is

$\psi(x, t) = A\,(\cos kx + \mathrm{i} \sin kx)\,(\cos \omega t - \mathrm{i} \sin \omega t)$.

We can imagine ψ as a helical waveform undulating in a complex space around the x-line. Schrödinger's equation then relates ψ to the energy of the system as coded by the Hamiltonian operator. Readable introductions to quantum mechanics that give the flavor of the math involved include those by Cox and Forshaw, by Susskind, and by Carroll. [CF 2012, SL 2014, CS 2024]

Page 64. Born's probability interpretation of the wavefunction is this: If the probability of a quantum event with wavefunction ψ is P, then $P = |\psi|^2 = \psi^* \psi$, where ψ^* is the complex conjugate of ψ.

Page 65. Dirac's classic and still widely read text introducing QED is great, but he wasn't a gifted communicator, and most beginners will prefer to start with the breezier introduction that Feynman wrote later, followed by a more modern text. [DP 1958, FR 1985]

Page 65. The renormalization problem for QED was independently solved by Feynman, Julian Schwinger, and Sin-Itiro Tomonaga, who shared the 1965 Nobel Prize in Physics for their work. Like so much in quantum theory, the math is too undisciplined to satisfy all mathematicians, but it seems to work in practice. Freeman Dyson proved the equivalence of the three approaches.

Page 66. Quantum field theory is notoriously hard to master, not least because the equations are a nightmare for people who prefer their math to be rigorous. Students will find a standard text like that by Schwartz too much to handle without interactive help. Zee's "simply as possible" introduction starts well, but halfway through it begins to run ragged. [SM 2014, ZA 2023a]

Page 67. Frank Close offers a useful and very short introduction to particle physics. [CF 2023b]

Page 67. The electromagnetic interaction and the weak nuclear interaction look very different, but in the hot early universe they were still a single electroweak interaction, as shown by Sheldon Glashow, Abdus Salam, and Steven Weinberg (Nobel Prize 1979). QCD describes nucleons and other hadrons as made of quarks and gluons, as proposed by Murray Gell-Mann (Nobel Prize 1969).

Page 67. The Higgs boson was first discovered in 2012 but proposed almost fifty years earlier by François Englert and Peter Higgs (who shared the Nobel Prize in 2013). The personal story behind it is expertly retold by Close. [CF 2023a]

Page 67. Wheeler's vivid account of general relativity for lay people is wonderful. Ferreira's summary of a century of secondary work surrounding it sets it in historical context. [WJ 1998, FP 2014]

Page 68. What Einstein said in the "dice" comment to Max Born is discussed in the biography by Abraham Pais, page 443. [PA 1982]

Page 68. Schrödinger published his cat story in an informal overview of quantum mechanics. The reverberations of the idea in later work have been endless. One such, a good one, is Gribbin's book, if you can get past the quirky title. [SE 1935, GJ 2025]

Page 69. Wigner aired his "Wigner's friend" scenario in 1961, and it was published in a 1962 anthology. More recently, the Frauchiger–Renner theorem derives a contradiction, under natural assumptions, from an extended Wigner scenario, which is harmless if the Born probabilities have a QBist interpretation. [WE 1962, FR 2018]

Page 69. Zeh was a professor in Heidelberg while I worked there. Full disclosure: I helped publish his book on the direction of time, now in its fourth edition. Lindley offers a highly readable account of the key idea. [ZH 1970, ZH 2001, LD 1997]

Page 70. Einstein's paper with Podolsky and Rosen has triggered a huge secondary literature, but Bell's later theorem, reprinted in his book, now overshadows all of it. [EP 1935, BJ 1987]

Page 72. Zeilinger has since published a light introductory book explaining his experimental work on testing Bell's theorem, entanglement, and teleportation. Aczel's account of entanglement and its back story is still good. [ZA 2023b, AA 2001]

Page 73. Carroll aired a perceptive view on the Copenhagen interpretation in his 2019 book. Freire's compendious collection covers the Copenhagen and many other interpretations of quantum theory. The "consistent histories" approach advocated by Omnès has now been incorporated in a "bubbles" approach reported by Gilligan-Lee. [CS 2019, FO 2022, HW 2000, OR 1999, GC 2025]

Page 75. Shannon first published his theory of information in a Bell Systems journal. His work has now been subsumed in algorithmic information theory, which merges it with computability theory. Chaitin's work in algorithmic information theory has given it deep conceptual foundations. [SC 1948, CG 1987]

Page 76. David Mermin published an interesting comment on quantum Bayesianism in the journal Nature. Freire's collection includes contributions on QBism. [MN 2014, FO 2022]

Page 79. The Heijenoort anthology includes three original papers by Brouwer, with useful commentary. Following a lot of work by numerous researchers on intuitionism, Michael Dummett wrote a helpful technical introduction. [HJ 1967, DM 1977]

Page 81. In standard notation, the classical equivalence is:

$\neg \forall x \; F(x) \dashv\vdash \exists x \neg F(x).$

The implication runs both from left to right and from right to left. For intuitionists, the problematic inference is from left to right:

$\neg \forall x \; F(x) \nvdash_{\text{int}} \exists x \neg F(x).$

Brouwer never formalized intuitionist logic, but his followers did. The source text I best recall is by Dummett. [DM 1977]

Page 81. As an example of such work, Per Martin-Löf presented early versions of his research on constructive math and computer programming while I was doing related work in Oxford. More generally, discrete math abjures the classical continuum and favors constructive methods. Wolfram has been a leading practitioner of discrete and computational math. [MP 1979, WS 2002]

Page 84. Quantum gravity is a huge and active research field, but popular introductions are also numerous. I especially enjoyed those by Rovelli and Carroll. For details of recent technical work, see the monograph by Wüthrich and Huggett, which covers the causal set theory, loop quantum gravity, and string theory approaches. [RC 2016b, CS 2019, WH 2025]

Page 84. Black hole research is also a huge and active field. Among the many recent books, Cox and Forshaw published a substantial introduction and Rovelli a delightful essay. [CF 2022, RC 2024]

Page 85. String theory really came alive for me in Greene's telling. When I then began to study Zwiebach's textbook, I found it was really hard. More recent work on applying string theory to black holes, reported by Samir Mathur, suggests it may be worthwhile. [GB 1999, ZB 2004, MS 2025b]

Page 85. The Maldacena–Susskind idea dubbed "ER = EPR" is spelled out in their arXiv preprint. Natalie Wolchover covers the idea in a more intelligible manner in the online magazine Quanta. [MS 2013, WN 2017]

Page 86. The Planck units based on c, G, $\hbar$, and also Boltzmann's constant k (to make a full set) combine these constants to build quantities that dimensionally match the usual SI units. They're good in theory, but they're too numerically extreme for everyday use:

Planck units: $l_P \sim 10^{-35}$ m, $t_P \sim 10^{-43}$ s, $m_P \sim 10^{-8}$ kg, $T_P \sim 10^{32}$ K

Frank Wilczek helpfully spelled out the Planck units twenty years ago in Physics Today. [WF 2005]

Page 86. On both Planck units and quantum foam, see the first pages of chapter 11 in Wheeler's autobiography. [WJ 1998]

Page 86. Elitzur and Dolev speculated on unfolding spacetime in section 17.10 of their *Quo Vadis Quantum Mechanics?* collection in a way that caught my imagination – and still does. Carroll described work on defining emergent spacetime via qubits in his 2019 book. [ED 2005, CS 2019]

Page 86. Fay Dowker, a pioneer in the "causal sets" approach to quantum gravity, says understanding the quantum gravity impasse requires a better understanding of time. [DF 2025]

LIFE

Page 88. Darwin's haiku is my own composition.

Page 89. Darwin's book is highly readable. Moorehead tells Darwin's story well, with beautiful illustrations. Dennett discusses the theory with masterful zest. [DC 1859, MA 1969, DD 1995]

Page 90. Mendel's story and the biology behind it are covered well enough in Wikipedia and other online sources.

Page 91. The chemical history behind the discovery of DNA and its structure are also covered well in Wikipedia and other online sources. Watson wrote the classic story of the discovery of the double helix structure of DNA. [WC 1953, WJ 1968]

Page 92. AlphaFold is a product of Google DeepMind. Google LLC is a multinational technology company based in California whose parent organization is Alphabet Inc.

Page 92. Lego bricks are products of the LEGO Group, a Danish construction toy production company.

Page 95. McFadden speculated that quantum effects might accelerate or coordinate evolution in his 2000 book but later went quiet on the idea. I like the idea but freely admit that neither of us know whether it could work in hard science. [MJ 2000]

Page 96. The anatomical facts about the human brain are easily accessible in standard texts or via online searches. I learned this year that the usually cited total of 86 billion neurons in the brain is apparently an example of false precision. [GA 2025]

Page 99. The main facts on how to monitor brainwaves or neural activity are again accessible in standard texts or via online searches.

Page 100. Neuralink Corporation was founded by Elon Musk and others in California in 2016. Neuropixels are a product developed in 2017 and manufactured by IMEC, an R&D organization with headquarters in Belgium. [ME 2019, PA 2022]

Page 101. Sejnowski et al. commented on how to make the best use of big data in neuroscience in Nature Neuroscience. See also the newer review by Dipietro et al. [SC 2014, DL 2023]

Page 101. The paper by Schlegel et al. describing their work on the fruit fly connectome appeared in Nature. [SP 2024]

Page 101. The paper by the MICrONS Consortium, referencing a dataset of high-resolution anatomical images of individual cells in mouse visual cortex that totaled 1.6 PB, was published in Nature. [MC 2025]

Page 101. The International Brain Laboratory conducted the study behind the map of mouse brain activity. [AB 2025, FH 2025]

Page 103. Singer's 2002 book offers a readable introduction in German to his own work on the brain. Singer and his group at the Max Planck Institute for Brain Research are still active, exploring the nonlinear dynamics of processing in neural networks linking coupled oscillatory circuits. [SW 2002]

Page 104. Llinás emphasized the importance of repeated thalamo-cortical looping for the sculpting of coherent thoughts in his 2001 book and at the 2002 meeting of the NYAS that I reviewed in an article reprinted in my book *Mindworlds*. [LR 2001, RA 2009]

Page 104. Edelman has written extensively on his work in neuroscience, which focuses on neuronal group selection in an approach sometimes called neural Darwinism. He co-authored his 2000 book with Tononi, who presented an early account of the integrated information theory. [EG 1992, ET 2000, EG 2004]

Page 104. The Chinese researchers' work on thalamo-cortical looping was reported in Nature. [MS 2025a]

Page 104. The spontaneous activity in the brain, its dekahertz rhythms, and how it constructs the world have all been described in detail by Buzsáki. [BG 2011, BG 2019, BG 2022]

Page 105. The importance of criticality for brain function has been emphasized in several recent studies reported by Robson in New Scientist. The 2020 paper relates criticality to the IIT discussed in my MINDS chapter. [RD 2025, PN 2020, MC 2024]

Page 105. Buzsáki discerns two distinct populations of neurons, fast ones for reflex responses and labile ones for learned behavior. This supports Kahneman's claim that our brains perform both System 1 and System 2 thinking. [BG 2019, BG 2022, KD 2011]

Page 106. Clark expresses his views on the predictive brain in his 2023 book in an accessible way. Anil Seth presents his own work on the predictive brain enjoyably in his 2021 book. Lisa Feldman Barrett glosses the idea authoritatively in one of her "seven and a half" lessons on the brain. [CA 2023, SA 2021, BL 2021]

Page 106. Evolutionary constraints limiting the veridicality of mindworlds are stressed by Hoffman, but in my opinion he exaggerates his case. [HD 2019]

Page 107. Clark and Chalmers presented their views on extended mind in a paper reprinted in the 2002 bible. [CD 2002]

Page 108. Dawkins aired his views in his 1976 and 1982 books, and they are well known. For my money, his best popular book on evolution is his 1990 bestseller *The Blind Watchmaker*.
[DR 1976, DR 1982, DR 1990]

Page 109. Susan Blackmore took up Dawkins' theme of memes in her 1999 book. She later wrote an introduction to consciousness that young general readers can enjoy. [BS 1999, BS 2003]

MINDS

Page 110. Chalmers' haiku is my own composition – and I'm not sure he'd approve of its last line. But he did work for his doctorate under Douglas Hofstadter, for whom I guess that line would sit quite comfortably.

Page 111. The 2023 Koch–Chalmers ceremony was reported in Scientific American. The results of the GWT–IIT runoff were published later in Nature. [HJ 2023, CC 2025]

Page 111. Crick and Koch shared credit for a 1990 article before they collaborated on the 1994 book. Chalmers' early works were already cited. In 2004, Koch published a superb introduction to the biology of consciousness, with lots of detail and good graphics.
[CK 1990, CF 1994, CD 1995, CD 1996, KC 2004]

Page 112. To recall where we stood a generation ago, read Steven Pinker for a long but readable account of the science and John Searle for some brief takes on the people involved, including Crick, Edelman, Roger Penrose, Dennett, Chalmers, and Israel Rosenfeld.
[PS 1997, SJ 1997]

Page 112. Panpsychism has been advocated for years by Galen Strawson, notably in JCS articles reprinted with others in 2024. Many researchers, including Chalmers and Koch, have expressed sympathy with forms of it, but no one has succeeded in working out its "cash value" in science. [SG 2006a, SG 2006b, SG 2024]

Page 113. Harris' short book refers to Strawson, also to her exchanges with Christof Koch. [HA 2019]

Page 113. I criticized McInnes' views, as recorded in his book and elsewhere, in my extended JCS review of his autobiography alongside other works. The review is in my *Mindworlds* collection. [MC 1999, RA 2009]

Page 114. Dennett's works span a busy lifetime in philosophy and include a fine collection he co-edited with Hofstadter, but the key one here is *Consciousness Explained.* [HD 1981, DD 1991]

Page 115. Nagel asked what it's like to be a bat in an article reprinted in the Chalmers bible. [CD 2002]

Page 115. Kurzweil describes the deployment of massive AI infrastructure as creating a digital neocortex for humanity. [KR 2012]

Page 116. Derek Parfit is the classic source on how to understand personal identity, though I'm inclined to dispute the ontological primacy of separate persons. See Part 3 of his book. [PD 1984]

Page 116. Interpersonal psychology is a huge theme that falls outside the scope of my book. Max Velmans has analyzed the 1P/2P contrast from a Kantian perspective in a way that mixes philosophy with psychology and neuroscience to advocate a position he calls reflexive monism. [VM 2000]

Page 118. Nagel described the view from nowhere in his 1986 book. It seems at first blush to reflect the objectivity of science, but I think it misrepresents the logical issue. [NT 1986]

Page 119. Baars presented his global workspace theory most notably in his elegant 1997 monograph. He probably sees that it's dated now and has recently cooled his claims for it. [BB 1997]

Page 121. Dennett said it's robots all the way down in New York in 2002. His article from that conference is in the proceedings volume edited by LeDoux. But his belief in the idea is evident from many of his published works. [LD 2003]

Page 121. Tononi presented a version of his IIT ideas in his 2000 book with Edelman. He presented a later formulation with Koch in a preprint in 2014. They and two others published a more final paper in Nature in 2016. In all its iterations, IIT looks better to practitioners than to thinkers. [ET 2000, TK 2014, TG 2016]

Page 122. Husserl's phenomenology is an arcane science with a vast literature. Husserl gave a useful presentation of it in 1929, which I haven't read, but it's hard to avoid the impression that its obscurity is a sign of conceptual confusion and scientific irrelevance. Despite that, some modern consciousness researchers have seen Husserl's work as a way of exploring experience – though even they tend to get lost in his idiosyncratic terminology. [HE 1929]

Page 122. Hume is one of the great British philosophers, and the secondary literature on his work is vast. Famously, his skepticism awakened Kant dramatically enough for Kant to respond by writing his classic critical trilogy. Hume's best words on causation are in a book reprinted in 2000. His basic idea is that our evidence for causation is "constant conjunction" at best – which we now express by saying correlation is not causation. [HD 2000]

Page 122. Pearl is an accomplished computer scientist who has worked for decades on modeling causality. His work is highly regarded and, so far as it goes, practically useful. But it relies on robust common sense about causality, which is precisely what Hume questioned. [PM 2018]

Page 125. Wittgenstein and Kripke expressed their views on language in the books previously cited. Wittgenstein's concerns are more general than Kripke acknowledges, so much so that more recent philosophers invented the fictional character Kripkenstein to represent the Wittgenstein that Kripke defended. The more general concerns range deep and wide in philosophy, but some are aired by Brandom in the course of arguing that Hegel already sensed them. Hegel understood that the meaning of any statement made here and now is always a hostage to fortune in future interpretations. Authors can only trust future readers to see sense in their words. [WL 1953, KS 1982, BR 2019]

Page 127. Descartes aired his pineal gland idea in his 1649 book, translated as *The Passions of the Soul*, relevant extracts from which are reprinted in the Chalmers bible. [CD 2002]

Page 127. Metzinger aired his views not only in his popular book but also in his earlier tome *Being No One*. I discussed them with him in 2009 and found we broadly agreed. [MT 2003, MT 2009]

Page 127. Libet's famous experiments showing conscious subjects making choices long after the firing of the relevant action potentials in the brain generated years of controversy before he published his book. They seemed to show action potentials arising in the brain several hundred milliseconds before a conscious choice to perform the relevant action. [LB 2004]

Page 128. A June 2025 report in Scientific American on biophotons titled "Your brain is glowing" should not confuse us here. The cited ultraweak photon emissions have been measured from many living organisms and result from relatively high-energy photons emitted by metabolic reactions in cells. Biophotons more widely appear to play essential roles in organisms and merit further research. [FC 2025, CH 2025]

Page 128. McFadden presented his CEMI theory in two 2002 articles, then teamed with Al-Khalili for their 2014 book. I discussed the theory with McFadden in 2003 and agreed with its general approach. [MJ 2002a, MJ 2002b, AM 2014]

Page 129. Zohar and Marshall outlined their views on BEC states in their 1990 book *The Quantum Self*. Zohar went on to write several books in a similar vein, but I gave up on them. [ZD 1990]

Page 130. Murphy presented her view of souls at the 2002 NYAS conference "The Self: From Soul to Brain." See her chapter in the LeDoux volume. The information-structure view of souls had appealed to me for years – and earned a brief mention in my 1996 sci-fi novel *Lifeball*. [LD 2003, RA 2012]

Page 132. On the duration of the specious present, the numerical check is this: For dekahertz photons, $f \sim 10$ Hz, so by $E = hf$, where $h \sim 10^{-33}$ J s, $E \sim 10^{-32}$ J; therefore, using $\Delta E\, \Delta t \sim h$, $\Delta t \sim 10^{-1}$ s, which is 100 milliseconds.

Page 132. Birds have been measured to have high flicker fusion frequencies. For three species of raptors, the measured rates were 81 Hz, 102 Hz and 129 Hz. [PS 2020]

Page 132. On the wavelength of dekahertz photons, $c = \lambda f$, where $c = 3 \times 10^{8}$ m s^{-1} and $f \sim 10$ Hz, so $\lambda \sim 10^{7}$ m, which amounts to thousands of kilometers.

WORLDS

Page 134. Neurath's haiku is my freely restyled rendering of an often-cited passage from his 1921 book *Anti-Spengler* (page 184): "Wie Schiffer sind wir, die auf offenem Meer ihr Schiff umbauen müssen, ohne je von unten frisch anfangen zu können. Wo ein Balken weggenommen wird, muß gleich ein neuer an die Stelle kommen, und dabei wird das übrige Schiff als Stütze verwendet. So kann das Schiff mit Hilfe der alten Balken und angetriebener Holzstücke vollständig neu gestaltet werden – aber nur durch allmählichen Umbau." [NO 1921]

Page 135. James' Edinburgh lectures on religious experience are hugely insightful from a psychological perspective. Essentially, his claim is that extreme religious experiences would often be taken as evidence for insanity in a clinical context. [JW 1902]

Page 137. Hofstadter reflects on minds reflecting other minds (at reduced resolution) in his memoir *I Am a Strange Loop*. His idea is that we each maintain little models of our friends and relatives that represent an extended being of their minds. [HD 2007]

Page 139. The Lewis modal logics S1–S5 are discussed in the classic textbook by Lewis and Langford and in the "new" (1996) one by Hughes and Cresswell. Popkorn discusses the Lewis systems together with Kripke's results and further work in a more advanced mathematical account of modal logic. The more recent literature on modal logics is vast. [LL 1959, HC 1996, PS 1994]

Page 139. Kripke's completeness result and his ideas in semantics appeared in the Journal of Symbolic Logic. Gödel published his mapping of the Lewis system S4 into intuitionistic logic in Vienna. For philosophical essays on Kripke's work in logic and semantics, see the Berger book. [KS 1959a, KS 1959b, GK 1932, BA 2011]

Page 140. Kripke presented his work on the semantics of intuitionistic logic at a colloquium in Oxford. [KS 1965]

Page 140. David Lewis' book on the logic of counterfactuals appeared when I was first studying Kripke's work in modal logic. He described how similarity rankings between possible worlds can clarify truth and meaning for counterfactuals. [LD 1973]

Page 141. Categories were first introduced decades ago by Saunders Mac Lane, but they remained an exotic topics for mathematicians. Eugenia Cheng recently brought them to life in a delightful book that goes from zero to research-level math. [MS 1998, CE 2023]

Page 142. Landauer published his ideas about information in the journal Physics Today. The idea has become integral to all current thinking about bits and qubits in physics. For example, Florian Neukart suggests that spacetime may be a quantum memory matrix, that this could solve the black hole information paradox, and that the mass of the qubits in spacetime could account for dark matter. [LR 1991, NF 2025]

Page 144. Chalmers introduces the VR idea at length in his light and readable book *Reality+*. The movie *The Matrix*, written and directed in 1999 by the Wachowskis, took center stage in Chalmers' essay "The Matrix as metaphysics" reprinted in his 2010 collection. [CD 2022, WT 1999, CD 2010]

Page 147. The size of the conjectured multiverse varies with the assumptions that lie behind it. Tegmark gives an excellent survey of the landscape of possibilities. Vilenkin gives a lively account of how our Big Bang bubble may have arisen in an inflationary ocean. Greene offers an insightful discussion of various ideas about the multiverse. [TM 2014, VA 2006, GB 2011]

Page 148. Penrose attempts in his big book *The Road to Reality* to cover everything important in math and physics. The outcome is often impressive, but his chapter (33) on twistors is wonderful. Twistors use "the magic of complex numbers" and ideas about category theory, quantum entanglement, and more. [PR 2004]

Page 149. Penrose presented his "orchestrated objective reduction" idea in his two intriguing books on consciousness. The physics of the idea is fairly conventional (certainly among theorists who study quantum measurement), but its application to consciousness – at least in the manner Hameroff proposes via microtubules – is deeply problematic, and hard for me to take seriously. Others have felt the same – see my essay "Roads to Reality" reprinted in *Mindworlds* – and Penrose has wisely gone silent on it. Yet research into the idea continued for years. [PR 1989, PR 1994, RA 2009]

Page 149. In the year Penrose published *The Emperor's New Mind*, Michael Lockwood published a book that covered many of the same themes from the standpoint of an Oxford philosopher. The contrast in styles was extreme, but their perplexity on the key issues was similar. [PR 1989, LM 1989]

Page 150. Quaternions were used to formulate special relativity by De Leo. Octonions have been used to elucidate the structure of the standard model of particle physics by Cohl Furey. An article about Furey by Natalie Wolchover appeared in the online journal Quanta. [DS 1995, FC 2014, WN 2018]

Page 150. My discussion of the sixth law is informed in part by the work of Ormrod and Barrett interpreting quantum theory in terms of causal bubbles, apparently to expel observers from the theory. For a light take, read Gilligan-Lee's report. [OB 2024, GC 2025]

Page 151. Wittgenstein's claim "I am my world" is the standard translation of proposition 5.63 in his *Tractatus Logico-Philosophicus*: "Ich bin meine Welt. (Der Mikrokosmos.)" He completed the tract under the title *Logisch-philosophische Abhandlung* in 1918. [WL 1998]

Page 152. The James Cameron movie *Avatar* raises fascinating philosophical questions of principle. Metzinger's VR ideas are fully consistent with the movie. [CJ 2009, MT 2003, MT 2009]

Page 153. My interpretation of the big self and little self follows a conversation I had with a participant (whose name I've forgotten) at the thirteenth ASSC conference, Berlin, 2009.

Page 154. Hofstadter's book is a classic. As I mentioned in a note to page 37, I first read it (with joy) in Japan in 1981. [HD 1979]

Page 155. The recursion of images between two parallel mirrors was a theme for both the younger and the older Hofstadter. It's a good image for recursive self-awareness. [HD 2007]

Page 156. A readable and reliable take on the neuroscience of free will is presented by Dan Wegener in his 2002 book. Dan Dennett aired the wider question of how free will can be given a reasonable interpretation in philosophy and concluded that a practical idea of freedom has evolved in human history in a way that Popper would have fit into his schema for science. [WD 2002, DD 2003]

Page 157. Debates about AI systems have taken a new turn since chatbots based on large language models have become so good at natural languages, but the basic point about computers not (yet) being conscious remains much as Penrose and Hofstadter in their different ways stated decades ago. The IIT claim by Tononi and Koch that even big AI systems have only tiny Φ measures tends to reinforce it – if you're prepared to accept their puzzling attempt to define Φ in mathematical terms and then apply it to AI hardware. [PR 1989, HD 1979, TK 2014]

Page 157. Turing's 1936 theorem solved what Davis later called the halting problem. Since then, the result has been raw material for Chaitin, who used it in his definition of the successive digits of his uncomputable number omega, Ω, which is thus algorithmically incompressible, hence random in the Chaitin–Kolmogorov sense. [TA 1936, DM 1958, CG 1987, CG 2005]

Page 157. All the math and science rolled out in this book suggests that humans can't predict their own future. Chalmers indulges the exercise of defying this claim in his 2012 book *Constructing the World*, where he considers whether we could infer all the truths about the world from a small initial set of truths, as a "homage" to Rudolf Carnap, who argued the case in 1928 – three years before Gödel's work torpedoed it. [CD 2012, CR 1928]

Page 158. Whether quantum computers might turn out to be conscious is an active topic at Google's Quantum AI lab, where its leader Hartmut Neven wants to entangle our brains with quantum processors to test the idea that consciousness involves quantum phenomena, reports New Scientist. [LT 2024]

Page 158. The technology of quantum computing is interesting but distracting. The logic and math behind it is more important for grasping its full implications. I learned something from the Nielsen and Chuang text (which is a ragbag of early ideas in the field), but later found that Bernhardt's brief monograph offers a simpler and cleaner introduction to the math. [NC 2000, BC 2019]

Page 159. Descartes actually said this first in French and then in Latin. His main relevant text is reprinted in English in Chalmers' collection. [CD 2002]

Page 160. Bold speculations on exotic higher life in the cosmos feature in Vidal's fascinating monograph. Vidal explores signs of life in black holes that eat their stellar partners in binary systems, but does so in a scientifically sound way (it's based on his doctoral thesis). Full disclosure: I helped to edit the text. [VC 2014]

Page 160. D. Graham Burnett discussed the idea of rocks doing math in the context of an insightful discussion of how AI will change the humanities. Wolfram speculates that the physics of rocks may be computationally universal. [BG 2025b, WS 2002]

Page 160. I republished the 1996 text of my novel *Lifeball* in 2012. But I now find much of it embarrassingly bad. [RA 2012]

Conclusion

Page 162. Hegel's haiku is my freely restyled translation of G.W.F. Hegel's famous sentence: "Wenn die Philosophie ihr Grau in Grau malt, dann ist eine Gestalt des Lebens alt geworden, und mit Grau in Grau lässt sie sich nicht verjüngen, sondern nur erkennen; die Eule der Minerva beginnt erst mit der einbrechenden Dämmerung ihren Flug." [HG 1821]

Page 163. What Goethe actually said was: "Es ist nicht genug zu wissen, man muss es auch anwenden. Es ist nicht genug zu wollen, man muss es auch tun." This is aphorism 324 in his maxims and reflections. [GJ 1833]

Page 164. Gödel's solution of Einstein's equation was noted on page 36. If asked, Gödel and Einstein would probably have agreed that the question of whether time is looped or not was an update of Kant's first antinomy of pure reason. Kant stated the antinomy in his critique of pure reason thus: "Die Welt hat einen Anfang in der Zeit, und ist dem Raum nach auch in Grenzen eingeschlossen. Die Welt hat keinen Anfang, und keine Grenzen im Raume, sondern ist, sowohl in Ansehung der Zeit, als des Raumes, unendlich."
[SP 1949, GK 1950, KI 1787]

Page 165. Nietzsche discussed eternal recurrence most famously in section 341 of his book *Die Fröhliche Wissenschaft* (a title usually translated as "The Gay Science"). [NF 1882]

Page 166. Ormrod and Barrett’s theory of quantum causal histories – “bubbles” – is detailed, technical, and rigorous. I haven’t digested it yet, but it seems to express exactly what Smolin may have meant loosely with his notion of “views.” [OB 2024, SL 2020]

Page 166. The question of how to avoid dismissing all religious discourse occupied me in my 2013 book *Coral.* Inspired by a 2010 German theological text on spiral dynamics, I discerned a neo-Hegelian thread in the historical evolution of religious ideas since the Axial Age that culminated in the rise of science as the better driver of such evolution in future. [KH 2010, RA 2013]

Page 166. Logical positivism began with the Vienna Circle around a hundred years ago. Gödel and Wittgenstein were among the early members of the circle. Eilenberger offers an account of its work. Alfred Ayer advocated its ideas clearly. [EW 2018, AA 1936]

Page 167. Pascal Boyer makes the anthropological case against religion by exploring some extraordinary pathologies in primitive belief systems. Dan Dennett has made a good case in favor of a continuing role for mainstream religion. [BP 2001, DD 2006]

Page 167. The doctrine of love recalls Mahatma Gandhi’s saying that the way of truth and love has always won. A humanist using my logic can accept that love is what best shapes our being, just as truth is what best organizes our existence.

Page 168. The best scientific biography of Einstein I know is by Pais. The best biography for the general reader is by Isaacson. Buchwald and Gordin’s new book on him is a brief take on the life. [PA 1982, IW 2017, BG 2025a]

Page 169. Spinoza deserves to be known more widely. Goldstein introduces him from a Jewish perspective. Damasio introduces him in the course of continuing his own good work in neuroscience. [GR 2006, DA 2003]

Page 169. I introduced the Globall in a Springer journal in 1991. [RA 1991]

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