About Time

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Abstract
This essay is an exercise in scientific metaphysics. Its aim is to sketch a unified account of time that both works in modern physics and makes sense in psychology. The raw materials for the sketch come from elementary logic and set theory. The experience of time flow is seen as a direct manifestation of a fundamental physical process. The ontology and epistemology of this experience can provide a foundation for psychology. If the physical sciences in their present form depict "the view from nowhere" onto reality, the new foundation can depict the view from anywhere.

Introduction

Time is arguably the most basic dimension of experience. Human consciousness seems conceivable only as a manifestation of temporal experience. Temporality implies ordered change, and a history of such changes is the autobiography of a conscious subject.

From a philosophical standpoint, the most fundamental task of an experiencing subject is to bring the manifold of phenomenal experience to a synthetic unity, and to do that on an ongoing basis by unifying the changing contents of the phenomenal manifold within the growing history of a uniquely located subject. This is essentially a Kantian view of the matter, but it remains viable as a one-sentence formulation of the metaphysics of rational subjectivity (where "rational" here means constrained by logic). When psychology is founded upon this basis, it presupposes a concept of time along with certain basic ontological concepts.

Time is normally regarded as a physical concept, and the physical view of time boasts not only great theoretical sophistication but also direct practical applicability. Yet the weakness in the physical concept of time is too basic and central to admit easy remedy. Despite centuries of progress in physics, the elementary experience of time flow remains unexplained. Indeed often, like Einstein, physicists feel free to regard the experience of time passing as a kind of illusion (Barbour 1999). On this classical and traditionally determinist view, physical reality is eternally as it is, and we experience change as a corollary of our limited perspective on that reality.

If we concede the primacy of physical time as it is conventionally understood even for the scientific explanation of changing experience, we admit the possibility in principle of detaching the subject from any anchor in a unique history. This follows from the basic philosophical perspective of the conventional physical sciences that they depict "the view from nowhere," which is to say they describe arbitrary domains of objects but not the subjects that reflect or comprehend them (Nagel 1986). Given a defined physical reality, the subject is inserted by hand, in an ad hoc manner. So a free-floating subject seems able in principle to roam around at will in the spacetime continuum, and this raises deep philosophical problems, for example about time travel. But most such ideas are based on naïve psychology.

Experienced time

The experience of time flow is basic to any phenomenology (Atmanspacher 1997). A subject has a past and a present, and faces a future. The unifying activity of the subject brings successive constellations of the phenomenal manifold to conceptual order as a series of momentary perspectives with timestamps that constitute a layered history of the subject. These successive phenomenal configurations undergo transition from the present to the past.

The subject confronts the future as a domain of possible forms of the phenomenal environment. In human beings, environmental configurations typically appear with affective coloration that disposes an individual to act so as to realize a specific form of the world within that domain of possible forms. An ongoing human world is thus made progressively more specific as a personal history accumulates (Velmans 2000).

Ontologically, the subject is embodied as a structure with a history. That history is a series of nested structures, each one of which represents the embodiment of the subject at an earlier time. Considered in terms of set theory, if each such structure is a set, with its own internal structure, then a history is an ordered series of sets, each one of which incorporates in some manner all of its predecessors in the series. Thus the history of a subject is the growth of such a series in time. New terms are added to the series as the subject brings successive initially future configurations of the phenomenal manifold to a synthetic unity.

Epistemologically, the subject recognizes ranges of phenomenal configurations as possible states of the world and selects specific states for realization in the present. The subject adds these states as new layers to an accumulating history. The process of bringing successive such selected configurations to a synthetic unity is epistemic, since it represents manipulations of concepts either within or beneath consciousness that have the effect of creating states of knowledge, and the study of that process is epistemological.

The temporal experience of a rational subject is thus reflected in an ongoing process with ontological and epistemological aspects. The process develops in time, and the subject grows a history. If we accept that the historical series of past states is part of the evolving subject whereas the domain of future possibilities is not, the experience of time is essentially asymmetric.

The problem for physicists who seek to understand temporal experience in terms of a physical concept of time is that time as a physical dimension seems to be symmetric between past and future (Flood 1986, Price 1996, Lockwood 2005). Arguably, all attempts to date to explain the asymmetry of time in terms of concepts like entropy have been inconclusive (Greene 2004). Yet the asymmetry is obvious to anyone who watches a movie run backwards, and given the persistence of memory is obvious to anyone who simply thinks rationally for a few seconds. This aspect of the dynamics of everyday phenomena and their binding in rational apperception is still a puzzle in contemporary physics. There seems to be a mismatch between the physical and the psychological concepts of time.

One response to this mismatch is to distinguish two concepts of time. Physical time remains symmetric and may be treated by strict analogy with a spatial dimension, as in relativity theory, where the transformation between time and the spatial dimensions is scaled by a physical constant known as the speed of light. Psychological time, by contrast, is asymmetric, with a fixed past and an open future. This asymmetry may be seen as a consequence of the ontological nature of the psychological subject, who acts in time to realize a growing and evolving self.

A fundamental question arises from the distinction between psychological and physical time. The two concepts must be deeply related. Psychological subjects inhabit a physical universe, so psychological time should be derivative in some way from an ontologically more basic physical time. Yet physicists are psychological subjects, and hence any conception they have of physical time is as a matter of biographical fact derived somehow from their experience of time. At the very least, we must require that physics and psychology, as scientific disciplines, together form a self-consistent account of the reality in which we find ourselves. So the two accounts of time must dovetail neatly together.

Physical time

In most of physics, time is regarded as a free parameter and is represented by a variable that may take arbitrary numerical values. For physics as conventionally understood on the basis of Einstein's work, time and space together form a four-dimensional continuum in which point-like events have coordinates consisting of ordered sets of four real numbers (Einstein 1922). Simultaneity is defined operationally in terms of the exchange of light signals, with the corollary that the prerelativistic concept of a single universal time coordinate for *now*, the present moment, becomes untenable.

Relativistically, events define four-dimensional light cones. Each event has a past light cone containing all other events that can possibly have had a causal influence, in principle via light signals, on the event. This past light cone contains small three-dimensional spatial volumes for the recent past and increasingly large volumes for increasingly remote times. And each event has a future light cone containing all possible future events that can be causally influenced by the event at its apex. The apical event is a sink for photons radiated from events in the past light cone and a source for photons radiating to events in the future light cone.

Curiously, light cones are bounded by “null infinities” for which time is unchanging. That is, the notional light rays that define the surface of a light cone form the locus of points that have zero temporal displacement from the apical event. The transformation equations for relativity imply that the proper time of an object, which is to say the time registered by a local clock for that object, dilates as the speed of that objects approaches light speed, as measured from an outside point. At light speed, the dilation becomes infinite and the local clock stops. Light rays are the traces of photons, and what this curiosity means is that time is nonexistent for a photon. If a photon were sentient, it would experience no passage of time at all between emission from a given material object, such as a star, and absorption in a faraway object, such as a human retina.

However, we cannot maintain that the spherical boundaries of the concentric spatial bubbles around an event in spacetime that touch the past and future light cones for that event define a set of events forming a relativistic analog of *now* for that event. That definition would extend the duration of *now* to infinity. Consider this example. At time zero, measured on a local clock, a photon is emitted toward to a remote mirror, where it is reflected back toward its source. Two years later, measured on the local clock, the photon is absorbed in an event with the same spatial coordinates as the emission event. The local time interval between the two events is thus two years, yet they are in the same *now* by the definition in terms of past and future light cones. This *reductio ad absurdum* applies to any events in the light cones. We need a better definition.

First, we define *world lines*. Events within the past and future light cones of an event are said to be separated by a timelike interval from that event. A pair of events with a timelike interval between them can be in causal interaction such that the earlier event may have a causal influence on the later event. By contrast, events outside the light cones are separated by a spacelike interval from the apical event, and are causally independent of that event. A world line is a set of events, each with a different time coordinate, such that all the intervals between the events are timelike and such that together they form either a continuous set (in the classical case) or a maximally dense set (in the case of discrete spacetime). That is, a world line is the possible history of a material particle.

Each event on a world line defines its own local *now*. And each event on a world line is related via possible or virtual photons with every other event on the world line. So *now* cannot be defined symmetrically in terms of photon exchange. But it can be defined asymmetrically. If each photon carries the timestamp of its source but not of its sink, then photons with timestamps in the past of a *now* event that are absorbed at that *now* event do not brand it with their old timestamp. And new photons emitted from the *now* event carry the *now* timestamp but do not stamp their absorption events with it. Their absorption events are in the future (even though the photons take no proper time to get there and hence still “think” the time is *now* when they get there). This definition implements a constructive metaphor according to which each momentary *now* is defined in terms of its past but not of its future, which has yet to be realized. Thus a world line is unidirectional, from past to future. Each event is defined by means of its past light cone, independently of its future light cone.

A neologism suggests itself here: a *world* is an event together with its past light cone. Thus a world line is either a continuous or a maximally dense series of worlds whose originating events form a timelike progression. Each world has a definite past but a wide range of possible or virtual futures. In this view, it makes no physical sense to say the future of a world is eternally there, just waiting to be discovered.

A world thus defined has a center, namely the event at the apex of its past light cone, and it has a definite *now*, namely the time coordinate of its apical event. A world thus defined is a relativistic invariant, since the causal history of an event (that is, the set of events that may in principle be in the causal ancestry of that event, namely the set of events in its past light cone) is the same to any observer of that event, in any state of (subluminal) motion relative to it. In everyday language, a world is a momentary “take” on the universe, centered at a specific spatiotemporal location.

The classical presumption that spacetime forms a continuum is thrown into question by quantum mechanics. The Heisenberg uncertainty relations between spatial location and momentum and between temporal location and energy limit the precision of possible measurements of the related items. Perfect precision in measurement of the spatiotemporal location of a particle would imply complete uncertainty in the momentum-energy measurement and vice versa. The minimum uncertainty in the product of the measurements is equal to the quantum of action known as Planck's constant, where an action is a spatiotemporally more or less localized quantity of momentum-energy.

The great discovery upon which quantum mechanics is based is that action is quantized. Relativistically, momentum and energy transform together like space and time, and energy is equivalent to mass, so what this means is that the smallest lumps of mass are spread out over a finite volume of spacetime. If the discrete elements of spacetime are sufficiently small to account for the facts, then it makes no empirical sense to insist that spacetime is continuous rather than discrete. Further, unless we can explain the infinities that would arise at singularities such as point masses or charges, it would make no theoretical sense to do so, either.

The quantum of action sets a specific scale to the granularity of spacetime. A photon has an energy proportional to the frequency of its corresponding electromagnetic wave. If there is a smallest physically possible increment of time, then there is a highest possible frequency, which specifies the largest possible quantum of energy. Similarly, a minimal spatial length corresponds to a maximal momentum in a given direction. Energy is equivalent to mass, and all mass exerts a mutual gravitational attraction. In general relativity, gravitation is a manifestation of the curvature of spacetime, as specified by Einstein's field equations, with a strength characterized by Newton's gravitational constant (Wheeler 1990). If we can mix quantum theory and general relativity in this way, a full circle of definitions and laws relates spacetime and mass using just three physical constants (for action, light speed, and gravitation). The measured values of the constants suggest that the ultimate granularity of spacetime is some twenty orders of magnitude below the diameter of a proton and some forty orders of magnitude briefer than a millisecond.

Accepting the discrete nature of spacetime seems to imply comprehending a finite number of worlds, where each world is a *now* event plus all the events in the past light cone of that event. Given the apparent fact that the universe has a finite age and the metaphysical claim that future events are possible but not (yet) actual, we can conjecture there are only finitely many actual worlds.

Quantum mechanics is centrally concerned with the collective behavior of events. Events are not spacetime points but nodes in a partial ordering, namely the causal or temporal ordering, which leaves some indeterminacy in the location of any entities they collectively realize. Collectively, events form structured setups or configurations called *systems* that instantiate *states*. A system can be something as simple as an elementary particle flying on some path from a source to a detector, and its state may be the specific path it takes, or its spin, or some other property. Predicting which states any given system will exhibit upon measurement is the main business of quantum mechanics (Feynman 1965, Shimony 1989, Lindsey 1996, Omnès 1999).

If past events are actualized whereas future events are no more than possible, any future discretely measurable state of a given system remains merely possible until such time as it either becomes or does not become actualized in the present as a result of performing a measurement. There is scope here for defining a special logic with a temporal modality, so that inferences to the past are classical but inferences to the future are probabilistic, but first we need to be clear on the physics.

Physical possibility is measured in terms of probability, and in quantum theory the mathematical tool for calculating probabilities is a complex function called the wave function. For a given system, the wave function specifies the evolution of the state of the system over time by encoding the respective probabilities of all its possible future states. There are two ways to calculate the probabilities of the future states of two or more systems, depending on whether the systems interact like particles or like waves. If the systems behave like particles and maintain their separate identities, their individual wave functions specify their respective probabilities to realize certain future states. But if the systems show wavelike behavior and interfere with each other, we need to add their wave functions before calculating the probabilities of the future states of the composite system.

Quantum theory raises puzzling questions about events. First, a quantum system can be in a *superposition* of multiple states. For example, a photon on its way from a source to a detector can be in a superposition of taking the left or the right path through a setup of mirrors, which is to say its behavior is wavelike despite its being an indivisible particle. This suggests that the events realizing the photon states do not yet suffice to determine a unique location for the photon. Second, there may be spatiotemporally extended *entanglements* between the parts of a quantum system. For example, a source may create two photons that fly off in opposite directions but remain entangled with each other so that any measurement of a property of one photon immediately determines the corresponding property of the other. This suggests that the events realizing the photon states do not yet suffice to determine separate locations for the two particles. Quantum superposition and entanglement are nonclassical notions.

Superposition of states suggests the coexistence of multiple realities, as possible or virtual states of a system, at least until the occurrence of an event that actualizes a unique state. A simple system (such as a single photon) may be in a *mixed* state with a nonzero probability to be in each of a number of unique states (such as photon spin states). A complex system (such as a number of photons) may be in a *coherent* state such that all its parts (such as photons) are in the same state. When a system in a coherent state interacts with another system, it may decohere into a mixture of different states. A complex system is generally in such a mixture of states, and the mixture evolves in time to blur into superposition any unique state that may arise. Given this idea, the most natural conception of the future is that it is a virtual (and not yet real) superposition of all possible outcomes.

Entanglement is a nonlocal phenomenon. The parts of a system in a coherent state are entangled with each other, and any interaction that changes the state of the system causes the entire system to change at once. The relative probabilities of states before and after such changes, both those predicted by quantum theory and those observed in experiments, are inconsistent with the view that they are caused by mechanisms operating subluminally (Bell 1987). Correlations exist between events with spacelike intervals between them, where we cannot invoke a classical, "local" mechanism to explain them. At the moment when two or more parts of a system become disentangled, the correlations between their states appear from a classical perspective to be sheer coincidences, even though quantum theory predicts them correctly. Typically, a new entanglement accompanies each disentanglement. For example, an entangled pair of photons from a distant star disentangles when one of them forms a new entanglement with atoms in the eye of a human observer.

It seems to be an anthropic fact that we see unique states and not superpositions of multiple states. If a system is in a mixed state prior to measurement and we then measure it, it is a fact about humans that we *now* see only a single state. We occupy a single state, and we entangle with the set of possible future states and thus change their probabilities relative to our own state, until the probability of one possible state becomes unity and the state is actualized for us. It is a corollary of our rational grasp of reality that we occupy a single extended state. At the quantum level, reality may dissolve into a foam of entanglements and mixed states, but as quasi-classical observers we do not experience the foam directly. Instead, we surf over it in a series of logical jumps until we have brought it to the synthetic unity of apperception. But this answer raises problems of its own. For example, do the apparently future states have any spatiotemporal location at all prior to our locating them in the *now* zone? Is everything beyond the *now* world merely virtual?

However, we may see unique states simply because they *are* unique. The wave function may undergo what Penrose calls objective reduction at a scale determined by quantum gravity (Callender 2001, Penrose 2004). His idea is that since superposed states coexist in a shared spacetime, any states that differ by an energy big enough to produce a quantum change in spacetime may trigger reduction of the wave function to realize a unique state. This proposal suggests that the transition from the quantum to the classical regime occurs at a scale characterized by the Planck energy (which is about twenty micrograms). The proposal is consistent with experiments conducted so far and is amenable to testing.

To predict the evolution of a physical world from *now* into the future, quantum theory suggests we should use the "sum over histories" approach pioneered by Feynman (Feynman 1985). We look at all the possible ways the world can evolve into the future, which is to say all the ways with nonzero probability, and add them up, or superpose them. The result is that a lot of states interfere destructively with each other, and thus cancel, and a lot of other states interfere constructively with each other and create extended entanglements. The path the world actually takes when it evolves from the *now* state into its next state is the path that minimizes the amount of action required to get there, in accordance with a quantum principle of least action. Naturally, this story glosses over a host of tricky technical details.

Mathematical time

Traditionally, mathematics is the study of eternal objects. Working mathematicians are often Platonists in the sense that they consider the realm of mathematical objects to be eternal and independent of human beliefs and choices (Benacerraf 1964). This traditional view is hard to maintain in an evolutionary epistemology. The fact that no reference is made to time in the specification of mathematical objects and the systems they form does not suffice to make them eternal. Yet the status of mathematical objects is quite different from that of physical objects. And one corollary of the traditional view is surely right: given certain axioms and rules, consistency requires that only certain theorems may be derived from them. Yet the choice of axioms and rules is in part arbitrary.

Following Frege's work on the foundations of mathematics, it has become increasingly clear that mathematics is logic plus certain ontological assumptions, which are typically expressed as axioms and rules in a more or less formalized syntax. Given an initial ontology, the practice of mathematics is then in large part a formalistic game in which truths relative to that ontology are made explicit through a process of computation, which is not a very creative process. The creative aspect of mathematics is to comprehend ontologies that turn out to be fruitful in practice.

The concept of time has a logic that requires a certain amount of ontological baggage, which at its minimum is already instantiated in a simple line of numbers. The elementary act of counting natural numbers is a good formal prototype for a temporal process. In this simplest logical model for time, there is a first moment (represented by the number zero or one) and time is discrete, but already there is scope for an infinite future and for a flow from future to past via the present moment (represented by the number currently in the counter). A first model of infinite divisibility is represented by the rational number line, and a model permitting arbitrary specification of time points is available with the real number line.

The process of doing mathematics may be seen as a psychological process. A subject manipulates tokens in time. The tokens represent elements of an ontology that appears to be eternal. From an evolutionary perspective, the ontology is the outcome of a temporal process with a quite different timescale from that of the manipulations of any given human subject. The stability of the various kinds of numbers seems to transcend that of the physical universe, yet because we denote mathematical objects with syntactic tokens that have contingent limitations, we cannot guarantee absolute stability. For example, the limitations of our syntax for natural numbers (say via binary sums of ascending powers of two) and the mechanisms we have at our disposal to manipulate that syntax (such as logic gates in electronic computers) set a probabilistic upper bound to the numbers we can reliably specify. Somewhere beyond the numbers we use, the landscape of numbers fades off into an indefinite blur.

This is relevant to the concept of time. The future is not necessary sharp as far out as we care to imagine. Indeed there is no reason at all why it should not become a blur surprisingly soon after the present moment. For example, just as predicting the exact future evolution of a microphysical system can become impossible beyond a small fraction of a second, so the future in general may simply be undefined until it becomes actualized in the present. Thus we may see our successive acts as creating not only forms in time but also the moments of time itself.

A mathematical metaphor exists to make this intuition more precise. The metaphor uses a mathematical tool that is about a century old and may soon rival in fruitfulness the number systems that have supported the growth of science since the time of Galileo. This tool is set theory, which as a formal axiomatic theory sits in a toolkit alongside more specialized tools such as topos theory and category theory. In brief, the metaphorical power of set theory is to replace "the view from nowhere" with the *view from anywhere*, in the following sense. We can use sets to articulate a new concept of time, to recast physics in a way that accommodates time flow, and to model the evolution of a rational subject.

Axiomatic set theory grew as a response to the paradoxes of naïve set theory. Naïve set theory was pioneered by Boole and Frege, and is based on the idea that any grouping of given elements, however specified, defines a set. Sets can be members of sets, and structures of sets can be arbitrarily complicated. Russell discovered the paradox that the set of all sets that are not members of themselves is a member of itself if and only if it is not a member of itself, and soon the whole of set theory was revised thoroughly to give it more rigorous foundations. The result, a century later, is a more or less widely accepted set of axioms, formulated in first-order logic, whose natural or intended model is a cumulative hierarchy based upon the null or empty set (Mendelson 1964). This hierarchy suffices as a formal ontology for practically all of mathematics.

The cumulative hierarchy is ontology at its most abstract. All its elements are ranked, starting with the null set at rank zero, continuing with its unit set at rank one, then with two sets at rank two, namely, first, the unit set of the unit set of the null set and, second, the pair set of the null set and its unit set, and so on to infinity. Each indexed level of the hierarchy accumulates all ranks up to and equal to the index of that level. At each level, the universal set for that level (which following von Neumann may be called the V-set for that level) is the set of all sets of lesser rank, and has as its subsets all sets of lesser or equal rank.

The key construction that generates the cumulative hierarchy is the power set construction. The power set of a given set is the set of all its subsets, including the null set and the given set itself. If all the sets so far are finite, the result is a potentially infinite cumulative hierarchy of hereditarily finite sets that suffices for a large body of practical mathematics, including that required to pursue the computational physics of a universe of finite worlds defined over a discrete spacetime.

Mathematics is sometimes defined as the science of the infinite. When sets become infinite, the going gets tougher. Cantor's great contribution was to see that if two infinite sets have the same cardinality when they can be mapped one-to-one to each other, then the cardinality of the power set of a given infinite set is strictly greater than the cardinality of the given set. Thus the cardinality of a countable set like the set of natural numbers is less than the cardinality of its power set, and so on. Hence there are infinitely many different infinite cardinalities and the cumulative hierarchy is a transfinite structure.

With this step, set theory became mathematically challenging, and no-one knows how far into the realm of transfinite sets it can be built up consistently. For example, we do not know whether or not the cardinality of the power set of a countable set is the *next* transfinite cardinality above the cardinality of a countable set. The assertion that it *is* the next is called the continuum hypothesis. What we do know is that if the hierarchy is built up in a constructive fashion, which is to say by building always from given elements, always making minimal existence assumptions, the result is Gödel's constructive hierarchy in which the continuum hypothesis holds (Jech 1971). We also know that the universe can be "forced" to disobey the continuum hypothesis (Cohen 1966). Numerous axiom systems exist, many of them agnostic on the continuum hypothesis, and the current favorite, ZF (Zermelo–Fraenkel) set theory, is just one among many options (Fraenkel 1958, Quine 1963).

Physicists who accept a discrete spacetime and regard information (and logic too) as related to physics are likely to question the need for transfinite set theory. But the concept of a limit, whereby an infinite continuation of a discrete process is taken as a conceptually completed construction, leads immediately to closed infinite sets and hence to the diagonal construction that generates Cantor's transfinite paradise. The concept of a limit is central to the infinitesimal calculus, and hence to complex analysis, which is still the main mathematical tool and foundation of theoretical physics. So ZF set theory is worth taking seriously.

In this picture, there is no ceiling at all to the cardinality of the universe of sets. Indeed the idea that there is a fixed universe of sets is paradoxical, since the universe would itself be a set, namely the set of all sets, and standard axiom systems such as ZF forbid any set from being a member of itself. In such systems, any attempt to define a universe of sets defines a V-set that is comprehended as an element in a more powerful theory, albeit a logically more perilous one.

It was a central plank of Quine's philosophy that any ontology may be mapped into an ontology of sets. The entities comprehended in any physical theory, or any scientific theory at all, if they are seen sufficiently abstractly, and if the theory is well defined and consistent, may be regarded as formally equivalent to the sets in some initial segment of the cumulative hierarchy (Quine 1960). The principle used here is that set theories with models in proper initial segments of the cumulative hierarchy are consistent. The difficulty is that *proving* their consistency requires the additional resources of set theories that can only be modeled in larger initial segments of the hierarchy. For theories whose smallest models are transfinite, proving their consistency requires theories whose own consistency is less assured than that of the theories for which they provide the putative proof.

This has consequences for a general theory of cognitive subjects in a physical universe. Any such subject can be represented logically by a suitable initial segment of the cumulative hierarchy. This may seem bizarre, but the idea is simple enough. A rational subject brings a cognitive landscape to a synthetic unity. If all the elements of that landscape are formally mapped to sets that have ranks below a certain level and hence are members of the V-set for that level, the subject may be mapped to that V-set. The growth of the subject into the future is then analogous to realization of successive V-sets, and the corresponding moves in time are analogous to counting through the indexes for the levels of the cumulative hierarchy.

The argument here works generally, both for the transfinite V-sets that believers in continuous spacetime need for their mathematical foundations and for the hereditarily finite V-sets that suffice for computational physics in discrete spacetime. But finite sets are much easier mathematically. Anyone impressed by Wolfram's advocacy of computational modeling as "a new kind of science" to replace the sciences based on the infinitesimal calculus and the metaphysics of the classical continuum will be happy to forget transfinite sets and work with finite worlds in the countable universe of discrete mathematics (Wolfram 2002).

Epistemological time

From an epistemological perspective, time is the dimension along which the knowledge of a rational subject evolves and in general grows. And as Popper pointed out, modern epistemology is the philosophy of science. Scientific knowledge does not simply accumulate but takes on new shapes that embrace what came before in new and often surprising ways. A temporal asymmetry is always evident. We can know past events but we cannot know future events. A structure of scientific knowledge is a logical structure (constrained to maximize consistency) in which knowledge of past events is put into some kind of order. If it is well adapted, such a structure need not be fundamentally revised every time new events are added to the growing past. The creative aspect of doing science is to adapt the structure of scientific theories in such a manner as to achieve precisely this stability in face of new facts.

The appearance of illusion in the passage of time that so impressed Einstein arises in part from the extent to which scientists have succeeded in achieving this stability in face of new facts. It appears as if the theorizing is already done and all that remains is to add details within the existing frame. But any major revolution in the history of science provides evidence that epistemology is not so simple. New vistas open out, and in the process old facts often gain new significance. Old facts can also fade into insignificance and even get lost, as a body of knowledge develops an independent life that survives robust encounters with awkward details.

This illustrates a feature of the temporal evolution of knowledge that merits careful attention. To model an evolving cognitive subject simply as a growing pile of V-sets is to ignore the revisionist and revolutionary aspects of new knowledge. A more indirect model is required. More carefully considered, a subject may be represented by a potentially arbitrary construction using the resources available within a V-set. Such a construction may represent the subject ignoring or forgetting aspects of previous experience, or reprocessing certain elements of experience so as to refashion the epistemic landscape. The formal resources of set theory allow any amount of such indirection. The temporal accretion of elements representing experience onto the growing pile of sets representing a developing history is analogous to the ongoing flow of input data to a computer. The computer may process that data in arbitrarily complicated ways to generate output that looks new and surprising.

Essential to the model of the temporal subject presented here is that a subject is defined by reference to the past and not the future. On the basis of a given past, the cognitive subject moves into the future and realizes new forms of subjectivity. This epistemic process is analogous to a physical process of growth or change, and we can describe it in terms of the fundamental process of symmetry breaking. Each momentary form of a subject remains embedded as a historical part of all possible futures of that subject. Logically, a future timeline includes possible future states of that subject if and only if it has an initial segment representing that history. The evolving subject represented by that initial segment has a symmetry with regard to those futures, which is broken when the subject evolves into a later form with a longer history, relative to which only a subset of those formerly possible futures remain possible. As time passes, the set of possible futures must be consistent with an ever longer and therefore more restrictive history.

To illustrate this point about symmetry by analogy, an object is more symmetrical when it looks the same from a wider range of perspectives, and an object can lose symmetry when it changes. For example, the symmetry of a ball is such that it looks the same from all angles, but it loses that symmetry when we paint a blob on it, since it now looks different from some angles. Similarly, a subject with a given history is consistent with a certain range of possible futures, but it loses consistency with part of that range when it evolves into a state with longer and more specific history. The additional historical facts are like the blob on the ball.

This issue is closely related to probability and entropy. A history may be encoded as information, and in general a longer history requires more information to encode it. Information is negentropy, so if more information is required to specify the state of a system then the entropy of that state is lower. If the different states of a system that are each specified by the same amount of information have equal probabilities to be realized, it follows that states with higher entropy have higher probabilities to be realized. Thus the epistemic state of a cognitive subject with a longer history, encoded as more information, has a lower entropy and hence a lower probability.

There is an apparent paradox here. As time passes, a system grows a longer history and hence become more improbable and loses entropy. Yet it is a central truth of thermodynamics that entropy tends to increase with time. To resolve the issue, we need to distinguish generic states from specific states, or in the terminology of thermodynamics, macrostates from microstates. Given a fully determined system, a generic state or macrostate of that system specified by less information is more probable than a less generic state specified by more information, because it is consistent with a larger number of fully specific states or microstates. If the system *itself* starts out in a relatively generic initial state with a short history and goes on to accumulate more "intrinsic" determinacy as it evolves in time toward a final specific state with a full history, then the later states of that system are more improbable. The earlier states of the system may be seen as more generic because they are consistent with a wider range of more specific future states, and the states of the system become less generic as time passes since more information is required to specify them. This is quite unlike the usual thermodynamic scenario, where an initial highly specified state evolves into a more *random* state that can be described approximately as a more generic state. So in fact there is no conflict with thermodynamics (Albert 2000, Zeh 1989).

Cosmological time

The account of time developed here is applicable quite generally, to cosmology no less than to the experience of a humble cognitive subject. The accounts of symmetry breaking and entropy apply unchanged on a universal scale, and carry the intriguing suggestion that the initial state of the universe, far from being maximally improbable as in the conventional modern story, was minimally informed and therefore maximally symmetric and maximally probable, but other aspects of the story invite further elucidation.

Any physical theory presupposes a background mathematical theory of some sort. Most physical theories are based on a fairly large body of mathematics, including number theory, complex analysis, and so on. Any such body of mathematics can be founded upon set theory, and more particularly upon the theory of some initial segment of the cumulative hierarchy such as the segment that serves as the natural model for ZF set theory. This is quite a lot to take on trust, and a philosophically well motivated cosmology would do well to start with less.

A logical cosmology can start with time and almost nothing else. As initial ontology it can take a single set, the universe, at level zero. This initial set is the null set, which falls into the first moment of time and thus becomes distinct from the universe, which is now determined as the level-one V-set whose only member is the null set. Given any existing levels filled with sets of given ranks, the universe is the superset of all the sets that can be built up from any of those ranked sets. Thus, by recursion, transfinite or otherwise, we build up a formal universe with a rich time dimension. This growing formal structure can be instantiated as a series of evolving physical entities in accordance with a standard big-bang story. In a story where spacetime is discrete and the total number of actualized worlds is always finite, the universe is always represented by a hereditarily finite V-set.

The special merit of this logical cosmology is that the mathematical entities do not exist in a shadow realm decoupled from physics, but arise with the physical universe in a cosmic bootstrap process. The bootstrap here is logical, and is required for a sequential account of cosmology that starts from a null description, independently of nature of the primordial state itself. The guiding idea here is that logic can only apply after the primal chaos reveals a first binary contrast. If spacetime is discrete, the total amount of information in any world with a finite history is always finite. It may be the case that logic cannot generate arbitrary levels of complexity in such worlds, but only complexity limited by the available information.

Essential to this story is that it is open-ended toward the future. Such a universe does not exist as a fixed formal structure, as in the “block” universe of classical relativity. It *cannot* exist as a closed structure, on pain of the sort of paradoxes that plagued naïve set theory a century ago. Models of time in an open-ended universe may represent closed futures with arbitrary levels of plausibility, but plausibility is not truth, and the only decisive test of any theory about a future state of affairs is to wait and see if that state of affairs is in fact realized. Scientists who favor a block universe are favoring a model that is ineluctably hypothetical.

Nevertheless, the open view of time is open to debate. Given the extremely exact and consistent corroboration of physical accounts of time in all areas of physics, we are strongly tempted to accept the conventional physical definition of time as absolute and abandon any remaining reservations as philosophical confusions. Time is what clocks measure and clocks are physical mechanisms, we are tempted to say, so belief in an open future and radical doubts about physical models are examples of prescientific thinking that merely betray lack of trust in physics.

Yet logic and mathematics tell a deeper story. Physics ignores the subject, and physical science articulates a view from nowhere. Logically, an appealing account of subjectivity is that subject and object are equal and opposite. Spelled out more fully, the rational subject of Kantian metaphysics is the reflective counterpart of the act of apperception that brings the manifold of phenomena to a synthetic unity. Each new realm of phenomenal content transforms the subject in a cognitive interaction that shapes both seen and seer. An ongoing subject is reflected in a series of momentary worlds that accumulate to build up a history. The transformations of this raw content in consciousness may be dramatic, so that the subjectively experienced result may bear little evident relation to any underlying physical process, but in principle the link remains as a tight coupling at some level. Views of the subject that lack such tight coupling run the risk of postulating a metaphysical ghost that drifts free of any plausible physical embodiment.

The mathematics of set theory can be deployed to back up this view of the subject. Any consistent universe of sets tops out in a V-set that reflects the universe so far. Yet we can count beyond any such V-set. In terms of time, any consistent view of the physical universe represents it at some time, as some specific world centered on a specific event, but time goes on and leaves that event behind. Any consistent view of the subject must be amenable to a similar logical argument.

In set theory, the topmost V-set as well as any sets with the same rank as that V-set are what set theorists call *proper classes* (Bernays 1958). Any set has two sides: seen from below it is a class and seen from above it is an element. Every set is the class of its members, and its members are elements. On this view, proper classes are just those sets that are not yet recognized as elements. Conversely, a proper element does not have a class side. In pure set theory there is only one proper element, the empty set. This complementarity of classes and elements in set theory is strictly analogous to that of subject and object in ontology.

The complementarity is reflected directly in natural language. A typical informative sentence has a subject–predicate form. And an easy way to motivate naïve set theory is to say that a set is the extension of a predicate, which is to say a set is the class of all elements that satisfy the predicate or for which the predicate is true. Thus a standard way to go about generating a semantics for natural language in set theory is to map a sentence in subject–predicate form to a formal statement asserting that the element or elements denoted by the subject of the sentence is a member or are members of the class representing the extension of the predicate.

We may readily go a step further and map the epistemic advance expressed by an informative sentence to a formal advance in the cumulative hierarchy from an initial level locating the element or elements denoted by the subject of the sentence to a final level locating the class denoted by the predicate. That is, the initial epistemic state required to provide a denotation for the subject of the sentence is still fuzzy in at least one regard, namely the regard specified in the predicate of the sentence, and the quantum of "defuzzification" provided by the sentence is precisely the difference between the initial and final epistemic states. In this story, both members of the pair of epistemic states are required to give a full semantic account of the informativeness of the sentence.

In physics, the open-ended cosmology suggested by set theory is currently under active technical investigation in connection with the quest for a consistent quantum theory of gravity. Conventional quantum theory presupposes a fixed background spacetime. Quantum field theory admits the equivalence of inertial frames defined by the Lorentz transformations and hence incorporates special relativity, but it still requires that distinct possible future distributions of mass-energy be regarded as superposing and interfering with each other in a single background spacetime. By contrast, general relativity requires that each distinct distribution of mass-energy corresponds to its own differently curved spacetime. In the classical picture of continuous spacetime, it is hard to see what criterion could be used to determine when spacetime branches into distinct configurations.

The *causal set theory* pursued by Sorkin and others represents a new approach to building a quantum theory of spacetime (Dowker 2003, Sorkin 2005). According to the causal set hypothesis, time is a process of becoming that corresponds to the continual birth of new elements of spacetime. In this approach, a discrete manifold embodies the metric relationships of spacetime and the structure of the manifold determines the temporal ordering of spacetime events.

A causal set (causet) is a discrete set of elementary events. A partial order is defined over the elements of a causet, where physically this ordering corresponds to the relation of before and after in time. In terms of causality, a given event is before another event if and only if the first event can in principle exert a causal influence on the second event (Sorkin 1991). A discrete world defined by a *now* event and its past light cone is an example of a causet.

This structure suffices to define a good approximation to the geometry of spacetime (Reid 1999). In general relativity, the local geometry of spacetime can be defined by ten numbers specified at each spacetime point. Nine of these numbers are determined by the light cone for each event. Knowing which events come before which others, as defined by the partial ordering, is equivalent to knowing the light cones. Thus if we know the causal ordering among events, we know nine of the ten numbers. The tenth number specifies spacetime volume. For causets, the spacetime volume of a region is simply the number of causet elements comprising that region.

The partial ordering that defines a causet is a pattern of ancestral relationships such that a given element is an ancestor of another element if and only if the first element temporally precedes the second. The development of a causet is a growth process in which elements are born one at a time. The growth starts from nothing, and each element that comes into being is born with a definite set of immediate ancestors. An element cannot be born before any of its ancestors, but otherwise the birth order is arbitrary, so different possible growth sequences are equivalent. In continuous spacetime, this equivalence corresponds to general covariance (which forbids *a priori* individuation of the points of a spacetime manifold as spatiotemporal events). For causets, it leads to a family of dynamics called classical sequential growth models. Work is ongoing to produce a quantum version of these models.

A successful prediction based on the causet view is that the finite stretch of time from the big bang to *now* corresponds to an energy uncertainty that is similar in size to recent empirical estimates of "dark energy" based on the accelerating expansion of the universe. This prediction suggests that the search for new sources of "dark energy" will fail to find any.

The puzzling issue of entanglement gets a new look in an open-ended cosmology. Spacetime grows discretely as new facts pop into existence. Measurements create new facts. Measurements that reveal nonclassical correlations between objects at separate locations can be interpreted as *creating* the separation. That is, the facts of such correlations first become local when measurements locate them. Two particles with a common origin retain a shared quantum state that causes a measurement of one of them to reveal a fact also about the other. The new interaction breaks the shared state and entangles the interacting particle with the measuring system – and also establishes a new spatiotemporal location for the interacting particle. The new location both breaks the correlation and creates the separation, and the apparent nonlocality of the correlation gets a natural explanation. This new look to an old and hard problem is encouraging for the causal set view of spacetime.

The general idea behind this new view of spacetime is that the *now* we experience as conscious subjects defines a wavefront in the growing cosmic network. The universe is only visible as a momentary world, with its own definition of *now*, and as the universe evolves around us we see a succession of worlds and a changing *now*. Metaphorically, our conscious lives surf on an advancing cosmic wavefront of crystallization marking the ongoing transition of possibilities to actualities.

Psychological time

The abstract and formalized rendition presented here of the subject is proposed as the central concept of a scientific psychology. To generalize Wittgenstein's assertion that "I am my world," the world is a world for a subject, and the idea of a world without a subject makes no sense (Wittgenstein 1922). The first subject is the knowing self, and that self is primordially coterminous with its world. Only later, with growing psychological sophistication, does a self learn that the world extends far beyond consciousness and includes other selves.

Correspondingly, in physics, only with the twentieth-century revolutions that superseded the solipsistic world view of Newtonian mechanics (with its absolute space and time implicitly reflecting a Kantian transcendental subject) has it become accepted that there is a plurality of worlds and that the time line is not a single rigid ruler for all purposes. The quantum revolution forced acceptance of possible future worlds, on a virtual par with each other, whose ongoing interaction generates a series of unique actual worlds.

Yet talk of worlds must be disciplined if the parallel and possible worlds of quantum theory are to be reconciled with the worlds of consciousness in Jamesian psychology (James 1902). The dated physical world defined by a causal nexus of events filling a past light cone represents a drastic transmutation and conceptual reduction compared with the worlds of human consciousness. In a fuller account, we need to recognize worlds at different levels. The set of photons that has impinged upon a given human being up to a given point in time is unique to that person, and this fact already ensures that the worlds of consciousness that collectively define a person are also unique to that person, but at a more macroscopic or generic level all humans share the same series of dated worlds. And given the highly adapted structures of our means for accommodating changes in time without having to rethink everything, at a yet more macroscopic or generic level we all inhabit a single shared world, or rather what we regard as a unique universe. Fine distinctions between microlevels and rough, pragmatic aggregations into macrolevels are among the everyday tools we deploy to bring different kinds and levels of unity to our variegated experiences. The stability of the background logic and physics for this picture offers no reason to deny its polymorphism at the psychological level (Ross 2004).

Each world is centered on an event. The instantiation of the world as a state of affairs is a fact, and given that fact, the local state at the apical event has probability one. The fixed event states in the past light cone for that event also have probability one, and the causal nexus they form may be seen at the appropriate macrolevel as entangling all those events into a single massive fact, which is equivalent to the conjunction of a vast number of atomic facts. As subjects centered on the apical event for a world, we entangle with its quantum spectrum of possible futures and realize a definite state, defined *post facto* as having probability one, which becomes the new world of our next incarnation. Thus we incarnate ourselves in a succession of momentary factual states, ordered along an epistemo-ontic dimension that we experience as time.

As human subjects, we do not experience the physically fundamental quanta of time directly. Our windows of specious present define a psychological *now* with a fuzzy duration of somewhere between many milliseconds and many seconds, depending on the granularity of *now* that best makes sense of our experience. This granularity is related to the size of the cycles of action and reaction that characterize our dealings as human agents, and is certainly remote by many orders of magnitude from the underlying quantum physics.

More generally, the specious present of human consciousness is the basis for all subjective apprehension of physical time spans. Crudely, any time span entertained in any way by a person corresponds via some symbolic mapping to a span of *now* in consciousness, and as a matter of logical principle any symbolic mapping is as good as any other for assigning a specific duration to *now*. For example, a person gazing into a starlit sky and reflecting on the cosmos may regard *now* as an era spanning billions of years. It is not obvious that this is any less realistic than measuring *now* in milliseconds, or in Planck intervals. In any case, the logic is the same. A landscape realizing some ontology is regarded as present, and is preceded by a past landscape. Ahead, possible future landscapes represent changes relative to the present. The envisaged sequence of landscapes is always a series of symbolic constructions.

Relatively recent research reveals that this series of symbolic constructions is realized or implemented in neurophysical processes with millisecond timescales of their own. Libet's discovery that those processes may begin hundreds of milliseconds before the conscious deliberations they implement suggests that the levels of symbolic coding between the first neural operations and the introspected results are built sequentially in a substantial computation that remains largely below the threshold of consciousness (Libet 2004). This in turn suggests that the symbolic unification achieved in the synthetic unity of conscious apperception is merely the tip of a computational iceberg.

To generalize the picture yet further, humans recognize *levels* of subjectivity. The shared subjectivity of human societies finds various representations in science and culture, for example as popular creation stories for the universe or as economic or religious models of how a society should be organized, but this level of subjectivity is generally widely separated from that of individual human beings living their everyday lives, who may have very personal perspectives and beliefs. A first recognition of this separation is deeply entrenched in the view of the self prevalent in societies where a universal deity represents an ultimate level of shared subjectivity. In such societies, it is by no means obvious how the personal self relates to the universal deity. Certainly, no simple mapping in set theory suggests itself. In general, then, hierarchies of subjects may be as complex as hierarchies of objects, and human subjects may be embedded deeply within such hierarchies.

Multiple levels of subjectivity are also deeply internalized within individual humans. Each of us is capable of adopting different perspectives to suit our circumstances, and in the process sliding up and down between levels from generic or cosmic to specific or personal, but each of us also holds at least two contrasting perspectives permanently in mind. Roughly, we each realize our own lived world from below or within and our own living self from above or outside.

We experience our own reality from within, cognitively or introspectively, when we internalize a model of an entire world, or in philosophical terms an *intentional* representation of the external world, as our apprehension of how things are or how reality seems to us (Chalmers 2002). This apprehension surfaces in consciousness but is partly unconscious. We project this model world onto the external world of our sensorimotor interaction without bearing constantly in mind that what seems to surround us is essentially a model that we have adapted over a lifetime of mostly unconscious fine-tuning to fit the ongoing stream of now mostly corroborative input from the senses.

We enact ourselves as if from the outside, like a puppet, within that model world. The puppet or analog self within that world is the projected self of our deliberations and rationalizations. If the inside view is analogous to an image of an external reality on a computer screen, the puppet self is like the blinking cursor on the screen, which is to say the focal point for willed action and change. The puppet self is the locus of feelings and emotions and the agent of actions and processes that are attributed to the self. The puppet self appears in consciousness as the self of self-consciousness.

These two apparently opposite views are in fact views of one and the same person, as closely matched as hand and glove. The view from within defines a standpoint or station and the view from outside defines the occupant of that station. There is an obvious set-theoretic analogy to a class and an element within that class. Indeed the adaptive fitting of self and world is precisely the iterative process of accommodation that generates an epistemo-ontic series of materialized V-sets.

To review the argument, time is the logical shadow of an epistemo-ontic process. Whether this shadow maps uniquely to the physical time of conventional theory is a secondary matter, which we may even regard as a criterion for evaluating the explanatory success of the underlying physics. The epistemo-ontic process in turn is the formal outline of a psychological process, which in human beings may best be regarded as taking the contingent form of ongoing evolutionary reprogramming within a cerebral neuronet that is coupled via sensorimotor interaction to a natural environment.

A world of consciousness is a cognitive structure implemented in a cerebral neuronet. It takes the form of an information structure that we can call a *virtual reality* reflecting more or less imperfectly via some more or less indirect symbolic mapping the facts surrounding that cognitive agent (Deutsch 1997). This virtual reality evolves more or less in lockstep with the external environment via the sensorimotor interaction that constitutes the life of the cognitive agent. The evolution is reflected as a series of momentary worlds, where each builds on its predecessors. This building process may be arbitrarily complicated and involve arbitrarily indirect relationships between layers, but all this is in principle capable of being modeled in a computer and therefore in discrete set theory. Each momentary form of the virtual reality is reflected in some V-set, where in general later forms will often be more complex and hence require larger V-sets. The picture that emerges would seem to invite implementation in robotic systems.

Recall that an axiom for this conception of consciousness is that at some level subject and object are equal and opposite. Ontologically, the most natural level for this complementarity is at rock bottom. In a discrete perspective based on quantum information theory, rock bottom may be the minimal qubits defining the elementary events that constitute reality. On this view, any sufficiently complex and dynamically interactive construction of qubits would instantiate subjectivity on a formal par with human subjectivity, and any particular state of human subjectivity would admit reduction without residue to a suitable constellation of qubits. This would imply that any finite subject may in principle be teleported as a stream of qubits, and thus travel to any future world containing a suitable "fax" machine. Conceptually, this is not so much time travel as suspended animation.

Returning to cosmology, the view that time is realized in the past and still virtual in the future concedes great ontological importance to the present. Physically, it accords a unique status to the apical event that defines a momentary world. Psychologically, given the primarily epistemo-ontic status of experienced time, it leaves open the possibility of radical novelty in the emergence of new worlds. If the cosmos is so far only determined up to *now* and the cloud of future possibilities must entangle with *now* to become bound into a definite past, then creation is ongoing and the cosmos is never complete.

Conclusion

Two concepts of time have been distinguished. Physical time is epistemologically flat in that it presupposes some more or less definite body of background physical theory, which may include special or general relativity, quantum decoherence and entanglement, and so on. Given such a background, time is defined independently of the experiences of cognitive agents. In effect, the experiencing subject of physical time is the correlate of a series of light-cone worlds. Psychological time records an epistemo-ontic process in which a self reflects a world in a succession of momentary configurations with an arbitrarily complicated logic. Psychologists may remain fairly agnostic on how that process is represented in physics. Current research indicates that in humans the process is embodied as an ongoing sensorimotor coupling with a virtual reality implemented in the cerebral neuronet.

The relation between these two views of time is obvious. Each presupposes the other. The present event of the light-cone view would make no sense if there were not some locally embodied observer at the *now* point to entangle with the possibilities and realize a definite factual landscape. And the explanation of human mental life in terms of brain states would make no sense without some detailed and specific background physics.

The metaphysical union of these two views may be called psychophysical. In principle, the phenomenology of time precedes its reduction to psychophysical science. In fact, phenomena seem to settle down by themselves into a specific spatiotemporal order and to bind of their own accord into the stable patterns that define us as psychic subjects interacting with physical objects. Defining time as a psychophysical category is thus rather natural.

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As precipitating cause for this essay in its present form, I cite recent work by Elitzur and Dolev:

> Suppose that there is indeed a 'now' front, on the one side of which there are past events, adding up as the 'now' progresses, while on its other side there are no events, and hence, according to Mach, not even spacetime. Spacetime thus 'grows' into the future as history unfolds …
>
> What role does the wave function play in this creation of new events? The dynamically evolving spacetime allows a radical possibility. Rather than conceiving of some empty spacetime within which the wave function evolves, the reverse may be the case: *The wave function evolves beyond the 'now', i.e., outside of spacetime, and its 'collapse' due to the interaction with other wave functions creates not only the events, but also the spacetime within which they are located in relation to one another* .

(Elitzur 2005, p. 346). I thank Springer editor Angela Lahee for related discussions.

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